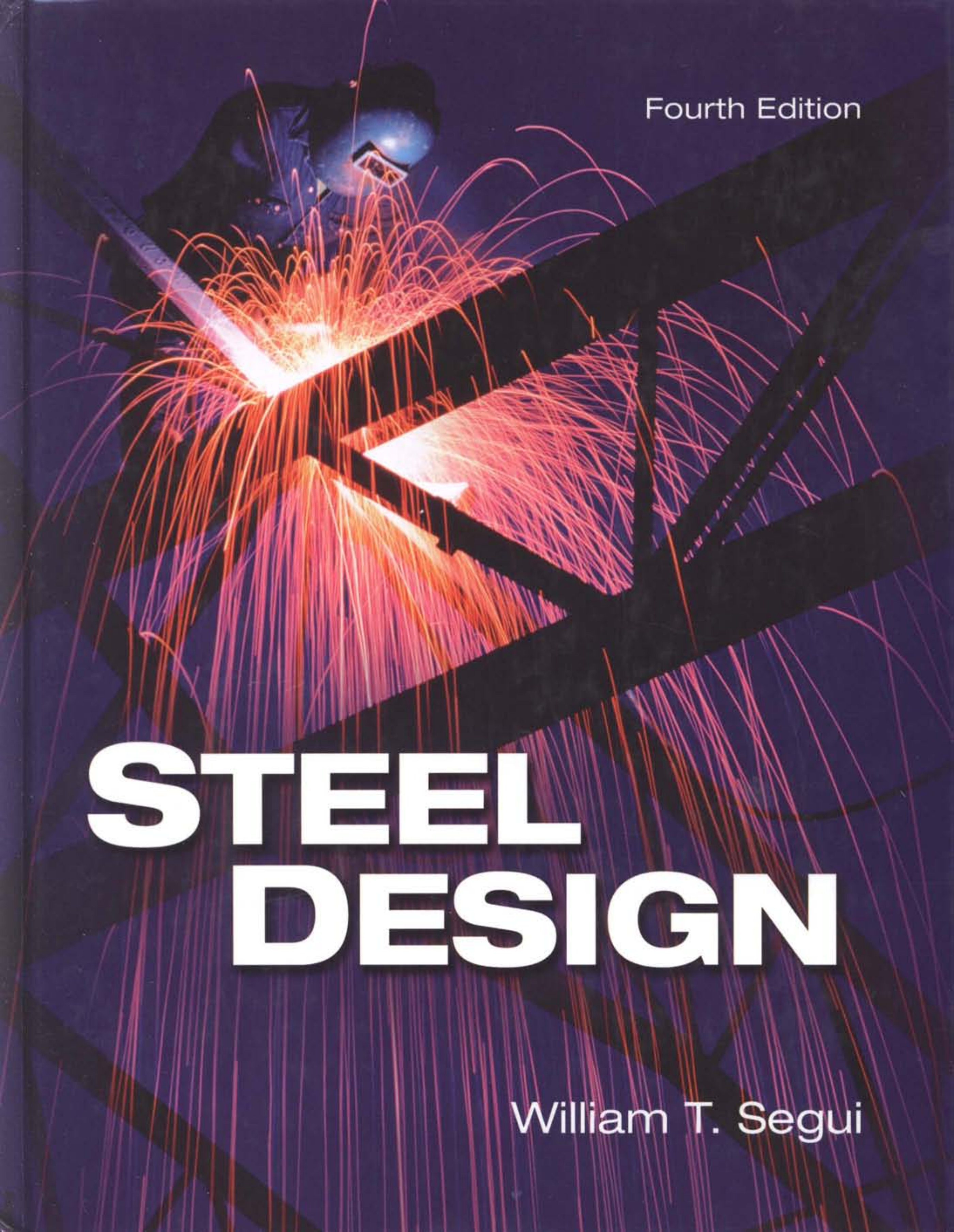


Fourth Edition

STEEL DESIGN

William T. Segui

4 Compression Members

4.1 DEFINITION

Compression members are structural elements that are subjected only to axial compressive forces; that is, the loads are applied along a longitudinal axis through the centroid of the member cross section, and the stress can be taken as $f = P/A$, where f is considered to be uniform over the entire cross section. This ideal state is never achieved in reality, however, because some eccentricity of the load is inevitable. Bending will result, but it can usually be regarded as secondary and can be neglected if the theoretical loading condition is closely approximated. Bending cannot be neglected if there is a *computed* bending moment. We consider situations of this type in Chapter 6, "Beam–Columns."

The most common type of compression member occurring in buildings and bridges is the *column*, a vertical member whose primary function is to support vertical loads. In many instances these members are also called upon to resist bending, and in these cases the member is a *beam–column*. Compression members are also used in trusses and as components of bracing systems. Smaller compression members not classified as columns are sometimes referred to as *struts*.

4.2 COLUMN THEORY

Consider the long, slender compression member shown in Figure 4.1a. If the axial load P is slowly applied, it will ultimately become large enough to cause the member to become unstable and assume the shape indicated by the dashed line. The member is said to have buckled, and the corresponding load is called the *critical buckling load*. If the member is stockier, as shown in Figure 4.1b, a larger load will be required to bring the member to the point of instability. For extremely stocky members, failure may occur by compressive yielding rather than buckling. Prior to failure, the compressive stress P/A will be uniform over the cross section at any point along the length, whether the failure is by yielding or by buckling. The load at which buckling occurs is a function of slenderness, and for very slender members this load could be quite small.

If the member is so slender (we give a precise definition of slenderness shortly) that the stress just before buckling is below the proportional limit — that is, the member is still elastic — the critical buckling load is given by

$$P_{cr} = \frac{\pi^2 EI}{L^2} \tag{4.1}$$

FIGURE 4.1

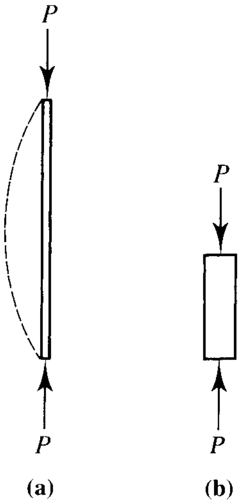
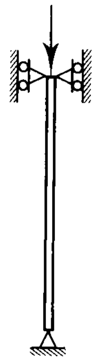


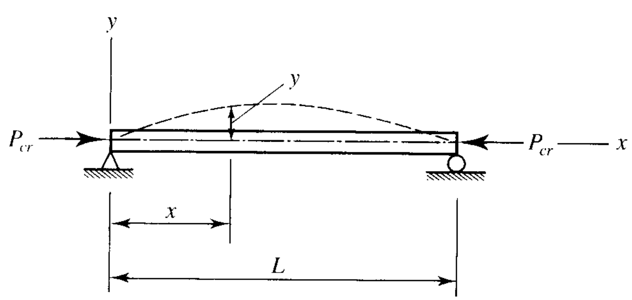
FIGURE 4.2



where E is the modulus of elasticity of the material, I is the moment of inertia of the cross-sectional area with respect to the minor principal axis, and L is the length of the member between points of support. For Equation 4.1 to be valid, the member must be elastic, and its ends must be free to rotate but not translate laterally. This end condition is satisfied by hinges or pins, as shown in Figure 4.2. This remarkable relationship was first formulated by Swiss mathematician Leonhard Euler and published in 1759. The critical load is sometimes referred to as the *Euler load* or the *Euler buckling load*. The validity of Equation 4.1 has been demonstrated convincingly by numerous tests. Its derivation is given here to illustrate the importance of the end conditions.

For convenience, in the following derivation, the member will be oriented with its longitudinal axis along the x-axis of the coordinate system given in Figure 4.3. The roller support is to be interpreted as restraining the member from translating either up or down. An axial compressive load is applied and gradually increased. If a temporary transverse load is applied so as to deflect the member into the shape indicated by the dashed line, the member will return to its original position when this temporary load is removed if the axial load is less than the critical buckling load. The critical buckling load, P_{cr}, is defined as the load that is just large enough to maintain the deflected shape when the temporary transverse load is removed.

FIGURE 4.3



The differential equation giving the deflected shape of an elastic member subjected to bending is

$$\frac{d^2y}{dx^2} = -\frac{M}{EI} \tag{4.2}$$

where x locates a point along the longitudinal axis of the member, y is the deflection of the axis at that point, and M is the bending moment at the point. E and I were previously defined, and here the moment of inertia I is with respect to the axis of bending (buckling). This equation was derived by Jacob Bernoulli and independently by Euler, who specialized it for the column buckling problem (Timoshenko, 1953). If we begin at the point of buckling, then from Figure 4.3 the bending moment is $P_{cr}y$. Equation 4.2 can then be written as

$$y'' + \frac{P_{cr}}{EI}y = 0$$

where the prime denotes differentiation with respect to x. This is a second-order, linear, ordinary differential equation with constant coefficients and has the solution

$$y = A\cos(cx) + B\sin(cx)$$

where

$$c = \sqrt{\frac{P_{cr}}{EI}}$$

and A and B are constants. These constants are evaluated by applying the following boundary conditions:

$$\text{At } x = 0,\ y = 0\text{:}\quad 0 = A\cos(0) + B\sin(0) \qquad A = 0$$

$$\text{At } x = L,\ y = 0\text{:}\quad 0 = B\sin(cL)$$

This last condition requires that $\sin(cL)$ be zero if B is not to be zero (the trivial solution, corresponding to $P = 0$). For $\sin(cL) = 0$,

$$cL = 0,\ \pi,\ 2\pi,\ 3\pi,\ \ldots = n\pi, \qquad n = 0, 1, 2, 3, \ldots$$

From

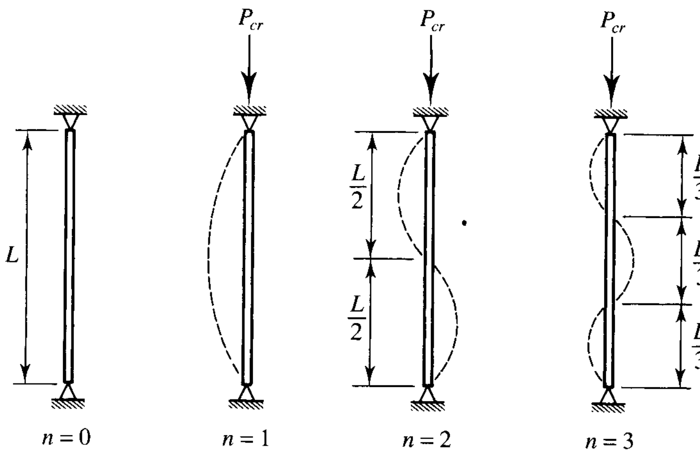
$$c = \sqrt{\frac{P_{cr}}{EI}}$$

we obtain

$$cL = \left(\sqrt{\frac{P_{cr}}{EI}}\right)L = n\pi, \quad \frac{P_{cr}}{EI}L^2 = n^2\pi^2 \quad \text{and} \quad P_{cr} = \frac{n^2\pi^2 EI}{L^2}$$

The various values of n correspond to different buckling modes; $n = 1$ represents the first mode, $n = 2$ the second, and so on. A value of zero gives the trivial case of no load. These buckling modes are illustrated in Figure 4.4. Values of n larger than 1 are

FIGURE 4.4



not possible unless the compression member is physically restrained from deflecting at the points where the reversal of curvature would occur.

The solution to the differential equation is therefore

$$y = B \sin\left(\frac{n\pi x}{L}\right)$$

and the coefficient B is indeterminate. This result is a consequence of approximations made in formulating the differential equation; a linear representation of a nonlinear phenomenon was used.

For the usual case of a compression member with no supports between its ends, $n = 1$ and the Euler equation is written as

$$P_{cr} = \frac{\pi^2 EI}{L^2} \tag{4.3}$$

It is convenient to rewrite Equation 4.3 as

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 EAr^2}{L^2} = \frac{\pi^2 EA}{(L/r)^2}$$

where A is the cross-sectional area and r is the radius of gyration with respect to the axis of buckling. The ratio L/r is the slenderness ratio and is the measure of a member's slenderness, with large values corresponding to slender members.

If the critical load is divided by the cross-sectional area, the critical buckling stress is obtained:

$$F_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E}{(L/r)^2} \tag{4.4}$$

At this compressive stress, buckling will occur about the axis corresponding to r. Buckling will take place as soon as the load reaches the value given by Equation 4.3, and the column will become unstable about the principal axis corresponding to the largest slenderness ratio. This axis usually is the axis with the smaller moment of inertia (we examine exceptions to this condition later). Thus the minimum moment of inertia and radius of gyration of the cross section should ordinarily be used in Equations 4.3 and 4.4.

Example 4.1 A W12 × 50 is used as a column to support an axial compressive load of 145 kips. The length is 20 feet, and the ends are pinned. Without regard to load or resistance factors, investigate this member for stability. (The grade of steel need not be known: The critical buckling load is a function of the modulus of elasticity, not the yield stress or ultimate tensile strength.)

Solution For a W12 × 50,

$$\text{Minimum } r = r_y = 1.96 \text{ in.}$$

$$\text{Maximum} \frac{L}{r} = \frac{20(12)}{1.96} = 122.4$$

$$P_{cr} = \frac{\pi^2 EA}{(L/r)^2} = \frac{\pi^2(29,000)(14.6)}{(122.4)^2} = 278.9 \text{ kips}$$

Answer Because the applied load of 145 kips is less than P_{cr}, the column remains stable and has an overall factor of safety against buckling of 278.9/145 = 1.92.

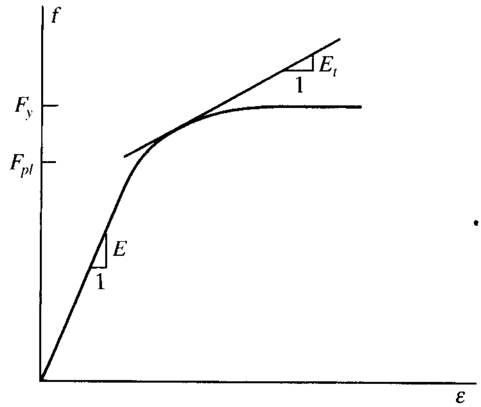
Early researchers soon found that Euler's equation did not give reliable results for stocky, or less slender, compression members. The reason is that the small slenderness ratio for members of this type causes a large buckling stress (from Equation 4.4). If the stress at which buckling occurs is greater than the proportional limit of the material, the relation between stress and strain is not linear, and the modulus of elasticity E can no longer be used. (In Example 4.1, the stress at buckling is P_{cr}/A = 278.9/14.6 = 19.10 ksi, which is well below the proportional limit for any grade of structural steel.) This difficulty was initially resolved by Friedrich Engesser, who proposed in 1889 the use of a variable tangent modulus, E_t, in Equation 4.3. For a material with a stress–strain curve like the one shown in Figure 4.5, E is not a constant for stresses greater than the proportional limit F_{pl}. The tangent modulus E_t is defined as the slope of the tangent to the stress–strain curve for values of f between F_{pl} and F_y. If the compressive stress at buckling, P_{cr}/A, is in this region, it can be shown that

$$P_{cr} = \frac{\pi^2 E_t I}{L^2} \tag{4.5}$$

Equation 4.5 is identical to the Euler equation, except that E_t is substituted for E.

The stress–strain curve shown in Figure 4.5 is different from those shown earlier for ductile steel (in Figures 1.3 and 1.4) because it has a pronounced region of nonlinearity. This curve is typical of a compression test of a short length of W-shape called a *stub column,* rather than the result of a tensile test. The nonlinearity is primarily because of the presence of residual stresses in the W-shape. When a hot-rolled shape cools after

FIGURE 4.5



rolling, all elements of the cross section do not cool at the same rate. The tips of the flanges, for example, cool faster than the junction of the flange and the web. This uneven cooling induces stresses that remain permanently. Other factors, such as welding and cold-bending to create curvature in a beam, can contribute to the residual stress, but the cooling process is its chief source.

Note that E_t is smaller than E and for the same L/r corresponds to a smaller critical load, P_{cr}. Because of the variability of E_t, computation of P_{cr} in the inelastic range by the use of Equation 4.5 is difficult. In general, a trial-and-error approach must be used, and a compressive stress–strain curve such as the one shown in Figure 4.5 must be used to determine E_t for trial values of P_{cr}. For this reason, most design specifications, including the AISC Specification, contain empirical formulas for inelastic columns.

Engesser's tangent modulus theory had its detractors, who pointed out several inconsistencies. Engesser was convinced by their arguments, and in 1895 he refined his theory to incorporate a reduced modulus, which has a value between E and E_t. Test results, however, always agreed more closely with the tangent modulus theory. Shanley (1947) resolved the apparent inconsistencies in the original theory, and today the tangent modulus formula, Equation 4.5, is accepted as the correct one for inelastic buckling. Although the load predicted by this equation is actually a lower bound on the true value of the critical load, the difference is slight (Bleich, 1952).

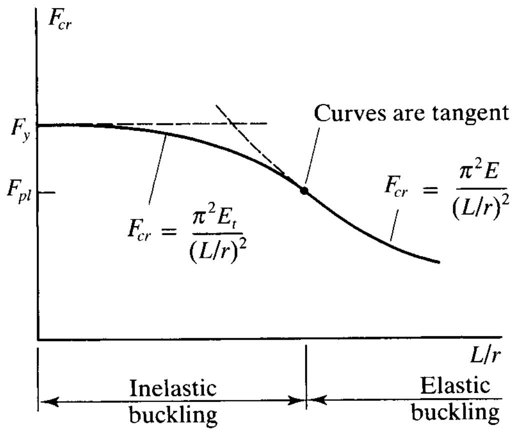
For any material, the critical buckling stress can be plotted as a function of slenderness, as shown in Figure 4.6. The tangent modulus curve is tangent to the Euler curve at the point corresponding to the proportional limit of the material. The composite curve, called a *column strength curve,* completely describes the strength of any column of a given material. Other than F_y, E, and E_t, which are properties of the material, the strength is a function only of the slenderness ratio.

Effective Length

Both the Euler and tangent modulus equations are based on the following assumptions:

1. The column is perfectly straight, with no initial crookedness.
2. The load is axial, with no eccentricity.
3. The column is pinned at both ends.

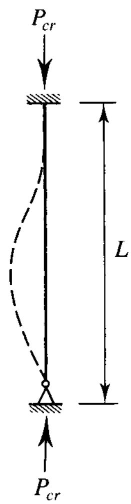
FIGURE 4.6



The first two conditions mean that there is no bending moment in the member before buckling. As mentioned previously, some accidental moment will be present, but in most cases it can be ignored. The requirement for pinned ends, however, is a serious limitation, and provisions must be made for other support conditions. The pinned-end condition requires that the member be restrained from lateral translation, but not rotation, at the ends. Constructing a frictionless pin connection is virtually impossible, so even this support condition can only be closely approximated at best. Obviously, all columns must be free to deform axially.

Other end conditions can be accounted for in the derivation of Equation 4.3. In general, the bending moment will be a function of x, resulting in a nonhomogeneous differential equation. The boundary conditions will be different from those in the original derivation, but the overall procedure will be the same. The form of the resulting equation for P_{cr} will also be the same. For example, consider a compression member pinned

FIGURE 4.7



at one end and fixed against rotation and translation at the other, as shown in Figure 4.7. The Euler equation for this case, derived in the same manner as Equation 4.3, is

$$P_{cr} = \frac{2.05\pi^2 EI}{L^2}$$

or

$$P_{cr} = \frac{2.05\pi^2 EA}{(L/r)^2} = \frac{\pi^2 EA}{(0.70L/r)^2}$$

Thus this compression member has the same load capacity as a column that is pinned at both ends and is only 70% as long as the given column. Similar expressions can be found for columns with other end conditions.

The column buckling problem can also be formulated in terms of a fourth-order differential equation instead of Equation 4.2. This proves to be convenient when dealing with boundary conditions other than pinned ends.

For convenience, the equations for critical buckling load will be written as

$$P_{cr} = \frac{\pi^2 EA}{(KL/r)^2} \quad \text{or} \quad P_{cr} = \frac{\pi^2 E_t A}{(KL/r)^2} \qquad \text{(4.6a/4.6b)}$$

where KL is the *effective length,* and K is the *effective length factor.* The effective length factor for the fixed-pinned compression member is 0.70. For the most favorable condition of both ends fixed against rotation and translation, $K = 0.5$. Values of K for these and other cases can be determined with the aid of Table C-C2.2 in the Commentary to the AISC Specification. The three conditions mentioned thus far are included, as well as some for which end translation is possible. Two values of K are given: a theoretical value and a recommended design value to be used when the ideal end condition is approximated. Hence, unless a "fixed" end is perfectly fixed, the more conservative design values are to be used. Only under the most extraordinary circumstances would the use of the theoretical values be justified. Note, however, that the theoretical and recommended design values are the same for conditions (d) and (f) in Commentary Table C-C2.2. The reason is that any deviation from a perfectly frictionless hinge or pin introduces rotational restraint and tends to reduce K. Therefore, use of the theoretical values in these two cases is conservative.

The use of the effective length KL in place of the actual length L in no way alters any of the relationships discussed so far. The column strength curve shown in Figure 4.6 is unchanged except for renaming the abscissa KL/r. The critical buckling stress corresponding to a given length, actual or effective, remains the same.

4.3 AISC REQUIREMENTS

The basic requirements for compression members are covered in Chapter E of the AISC Specification. The nominal compressive strength is

$$P_n = F_{cr}A_g \qquad \text{(AISC Equation E3-1)}$$

For LRFD,

$$P_u \leq \phi_c P_n$$

where

P_u = sum of the factored loads
ϕ_c = resistance factor for compression = 0.90
$\phi_c P_n$ = design compressive strength

For ASD,

$$P_a \leq \frac{P_n}{\Omega_c}$$

where

P_a = sum of the service loads
Ω_c = safety factor for compression = 1.67
P_n/Ω_c = allowable compressive strength

If an allowable stress formulation is used,

$$f_a \leq F_a$$

where

f_a = computed axial compressive stress = P_a/A_g
F_a = allowable axial compressive stress

$$= \frac{F_{cr}}{\Omega_c} = \frac{F_{cr}}{1.67} = 0.6F_{cr} \tag{4.7}$$

In order to present the AISC expressions for the critical stress F_{cr}, we first define the Euler load as

$$P_e = \frac{\pi^2 EA}{(KL/r)^2}$$

This is the critical buckling load according to the Euler equation. The Euler stress is

$$F_e = \frac{P_e}{A} = \frac{\pi^2 E}{(KL/r)^2} \qquad \text{(AISC Equation E3-4)}$$

With a slight modification, this expression will be used for the critical stress in the elastic range. To obtain the critical stress for elastic columns, the Euler stress is reduced as follows to account for the effects of initial crookedness:

$$F_{cr} = 0.877F_e \tag{4.8}$$

For inelastic columns, the tangent modulus equation, Equation 4.6b, is replaced by the exponential equation

$$F_{cr} = \left(0.658^{\frac{F_y}{F_e}}\right)F_y \tag{4.9}$$

With Equation 4.9, a direct solution for inelastic columns can be obtained, avoiding the trial-and-error approach inherent in the use of the tangent modulus equation. At the boundary between inelastic and elastic columns, Equations 4.8 and 4.9 give the same value of F_{cr}. This occurs when KL/r is approximately

$$4.71\sqrt{\frac{E}{F_y}}$$

To summarize,

$$\text{When } \frac{KL}{r} \le 4.71\sqrt{\frac{E}{F_y}}, \quad F_{cr} = (0.658^{F_y/F_e})F_y \tag{4.10}$$

$$\text{When } \frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}}, \quad F_{cr} = 0.877F_e \tag{4.11}$$

The AISC Specification provides for separating inelastic and elastic behavior based on either the value of KL/r (as in equations 4.10 and 4.11) or the value of F_e. The limiting value of F_e can be derived as follows. From AISC Equation E3-4,

$$\frac{KL}{r} = \sqrt{\frac{\pi^2 E}{F_e}}$$

For $\frac{KL}{r} \le 4.71\sqrt{\frac{E}{F_y}}$,

$$\sqrt{\frac{\pi^2 E}{F_e}} \le 4.71\sqrt{\frac{E}{F_y}}$$

$$F_e \ge 0.44F_y$$

The complete AISC Specification for compressive strength is as follows:

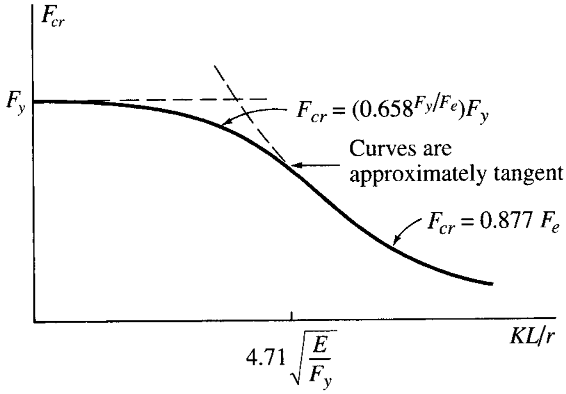
$$\text{When } \frac{KL}{r} \le 4.71\sqrt{\frac{E}{F_y}} \quad \text{or} \quad F_e \ge 0.44F_y,$$

$$F_{cr} = (0.658^{F_y/F_e})F_y \qquad \text{(AISC Equation E3-2)}$$

$$\text{When } \frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}} \quad \text{or} \quad F_e \ge 0.44F_y, a$$

$$F_{cr} = 0.877F_e \qquad \text{(AISC Equation E3-3)}$$

FIGURE 4.8



In this book, we will usually use the limit on KL/r, as expressed in Equations 4.10 and 4.11.

These requirements are represented graphically in Figure 4.8.

AISC Equations E3-2 and E3-3 are a condensed version of five equations that cover five ranges of KL/r (Galambos, 1988). These equations are based on experimental and theoretical studies that account for the effects of residual stresses and an initial out-of-straightness of $L/1500$, where L is the member length. A complete derivation of these equations is given by Tide (2001).

Although AISC does not require an upper limit on the slenderness ratio KL/r, an upper limit of 200 is recommended (see user note in AISC E2). This is a practical upper limit, because compression members that are any more slender will have little strength and will not be economical.

Example 4.2 A W14 × 74 of A992 steel has a length of 20 feet and pinned ends. Compute the design compressive strength for LRFD and the allowable compressive strength for ASD.

Solution Slenderness ratio:

$$\text{Maximum } \frac{KL}{r} = \frac{KL}{r_y} = \frac{1.0(20 \times 12)}{2.48} = 96.77 < 200 \quad \text{(OK)}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113$$

Since 96.77 < 113, use AISC Equation E3-2.

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(96.77)^2} = 30.56 \text{ ksi}$$

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(50/30.56)}(50) = 25.21 \text{ ksi}$$

The nominal strength is

$$P_n = F_{cr}A_g = 25.21(21.8) = 549.6 \text{ kips}$$

LRFD Solution The design strength is

$$\phi_c P_n = 0.90(549.6) = 495 \text{ kips}$$

ASD Solution From Equation 4.7, the allowable stress is

$$F_a = 0.6F_{cr} = 0.6(25.21) = 15.13 \text{ ksi}$$

The allowable strength is

$$F_a A_g = 15.13(21.8) = 330 \text{ kips}$$

Answer Design compressive strength = 495 kips. Allowable compressive strength = 330 kips.

In Example 4.2, $r_y < r_x$, and there is excess strength in the x-direction. Square structural tubes (HSS) are efficient shapes for compression members because $r_y = r_x$ and the strength is the same for both axes. Hollow circular shapes are sometimes used as compression members for the same reason.

The mode of failure considered so far is referred to as *flexural* buckling, as the member is subjected to flexure, or bending, when it becomes unstable. For some cross-sectional configurations, the member will fail by twisting (torsional buckling) or by a combination of twisting and bending (flexural-torsional buckling). We consider these infrequent cases in Section 4.8.

4.4 LOCAL STABILITY

The strength corresponding to any buckling mode cannot be developed, however, if the elements of the cross section are so thin that *local* buckling occurs. This type of instability is a localized buckling or wrinkling at an isolated location. If it occurs, the cross section is no longer fully effective, and the member has failed. I- and H-shaped cross sections with thin flanges or webs are susceptible to this phenomenon, and their use should be avoided whenever possible. Otherwise, the compressive strength given by AISC Equations E3-2 and E3-3 must be reduced. The measure of this susceptibility is the width–thickness ratio of each cross-sectional element. Two types of elements must be considered: unstiffened elements, which are unsupported along one edge parallel to the direction of load, and stiffened elements, which are supported along both edges.

Limiting values of width–thickness ratios are given in AISC B4, "Classification of Sections for Local Buckling," where cross-sectional shapes are classified as *compact, noncompact,* or *slender,* according to the values of the ratios. For uniformly compressed elements, as in an axially loaded compression member, the strength must be reduced if the shape has any slender elements. The width–thickness ratio is given the generic name of λ. Depending on the particular cross-sectional element, λ for I- and H-shapes is either the ratio b/t or h/t_w, both of which are defined presently. If λ is greater than the specified limit, denoted λ_r, the shape is slender, and the potential for local buckling must be accounted for. (We postpone a discussion of the compact and noncompact categories until Chapter 5, "Beams.") For I- and H-shapes, the projecting flange is considered to be an unstiffened element, and its width can be taken as half the full nominal width. Using AISC notation gives

$$\lambda = \frac{b}{t} = \frac{b_f/2}{t_f} = \frac{b_f}{2t_f}$$

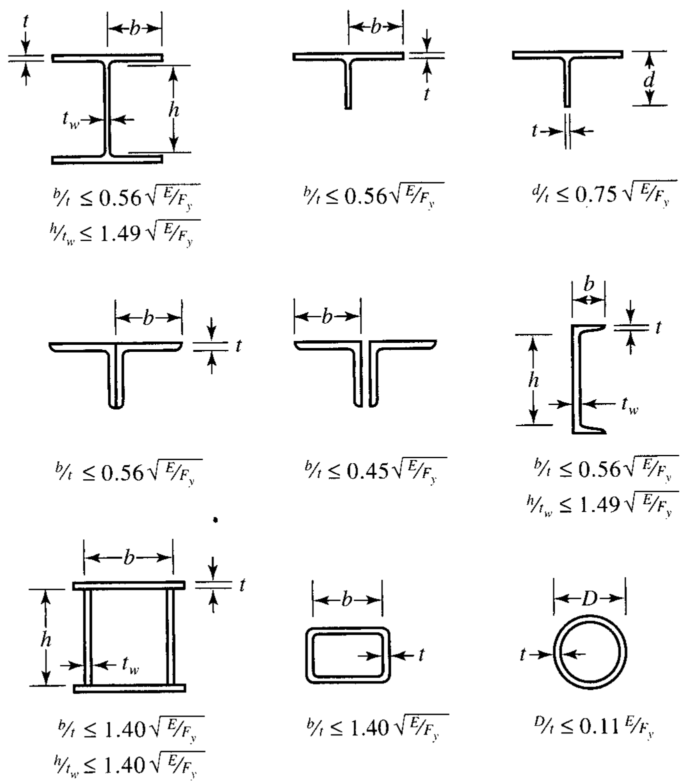
where b_f and t_f are the width and thickness of the flange. The upper limit is

$$\lambda_r = 0.56\sqrt{\frac{E}{F_y}}$$

The webs of I- and H-shapes are stiffened elements, and the stiffened width is the distance between the roots of the flanges. The width–thickness parameter is

$$\lambda = \frac{h}{t_w}$$

FIGURE 4.9



where h is the distance between the roots of the flanges, and t_w is the web thickness. The upper limit is

$$\lambda_r = 1.49\frac{E}{\sqrt{F_y}}$$

Stiffened and unstiffened elements of various cross-sectional shapes are illustrated in Figure 4.9. The appropriate compression member limit, λ_r, from AISC B4 is given for each case.

Example 4.3 Investigate the column of Example 4.2 for local stability.

Solution For a W14×74, $b_f = 10.1$ in., $t_f = 0.785$ in., and

$$\frac{b_f}{2t_f} = \frac{10.1}{2(0.785)} = 6.43$$

$$0.56\sqrt{\frac{E}{F_y}} = 0.56\sqrt{\frac{29,000}{50}} = 13.5 > 6.43 \qquad \text{(OK)}$$

$$\frac{h}{t_w} = \frac{d - 2k_{des}}{t_w} = \frac{14.2 - 2(1.38)}{0.450} = 25.4$$

where k_{des} is the *design* value of k. (Different manufacturers will produce this shape with different values of k. The *design* value is the smallest of these values. The *detailing* value is the largest.)

$$1.49\sqrt{\frac{E}{F_y}} = 1.49\sqrt{\frac{29,000}{50}} = 35.9 > 25.4 \qquad \text{(OK)}$$

Answer Local instability is not a problem.

In Example 4.3, the width-thickness ratios $b_f/2t_f$ and h/t_w were computed. This is not necessary, however, because these ratios are tabulated in the dimensions and properties table. In addition, shapes that are slender for compression are indicated with a footnote (footnote c).

It is permissible to use a cross-sectional shape that does not satisfy the width-thickness ratio requirements, but such a member may not be permitted to carry as large a load as one that does satisfy the requirements. In other words, the strength could be reduced because of local buckling. The overall procedure for making this investigation is as follows.

- If the width-thickness ratio λ is greater than λ_r, use the provisions of AISC E7 and compute a reduction factor Q.
- Compute KL/r and F_e as usual.
- If $\frac{KL}{r} \leq 4.71\sqrt{\frac{E}{QF_y}}$ or $F_e \geq 0.44QF_y$,

$$F_{cr} = Q\left(0.658^{\frac{QF_y}{F_e}}\right)F_y \qquad \text{(AISC Equation E7-2)}$$

- If $\frac{KL}{r} > 4.71\sqrt{\frac{E}{QF_y}}$ or $F_e < 0.44QF_y$,

$$F_{cr} = 0.877F_e \qquad \text{(AISC Equation E7-3)}$$

- The nominal strength is $P_n = F_{cr}A_g$ (AISC Equation E7-1)

The reduction factor Q is the product of two factors—Q_s for unstiffened elements and Q_a for stiffened elements. If the shape has no slender unstiffened elements, $Q_s = 1.0$. If the shape has no slender stiffened elements, $Q_a = 1.0$.

Many of the shapes commonly used as columns are not slender, and the reduction will not be needed. This includes most (but not all) W-shapes. However, a large number of hollow structural shapes (HSS), double angles, and tees have slender elements.

AISC Specification Section E7.1 gives the procedure for calculating Q_s for slender unstiffened elements. The procedure is straightforward, and involves comparing the width-thickness ratio with a limiting value and then computing Q_s from an expression that is a function of the width-thickness ratio, F_y, and E.

The computation of Q_a for slender stiffened elements is given in AISC E7.2 and is slightly more complicated than the procedure for unstiffened elements. The general procedure is as follows.

- Compute an effective area of the cross section. This requires a knowledge of the stress in the effective area, so iteration is required. The Specification allows a simplifying assumption, however, so iteration can be avoided.
- Compute $Q_a = A_{eff}/A$, where A_{eff} is the effective area and A is the actual area.

The details of the computation of Q_s and Q_a will not be given here but will be shown in the following example, which illustrates the procedure for an HSS.

Example 4.4 Determine the axial compressive strength of an HSS 8 × 4 × ⅛ with an effective length of 15 feet with respect to each principal axis. Use $F_y = 46$ ksi.

Solution Compute the overall, or flexural, buckling strength.

$$\text{Maximum } \frac{KL}{r} = \frac{KL}{r_y} = \frac{15 \times 12}{1.71} = 105.3 < 200 \qquad \text{(OK)}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29,000}{46}} = 118$$

Since 105.3 < 118, use AISC Equation E3-2.

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29,000)}{(105.3)^2} = 25.81 \text{ ksi}$$

$$F_{cr} = 0.658^{(F_y/F_e)} F_y = 0.658^{(46/25.81)}(46) = 21.82 \text{ ksi}$$

The nominal strength is

$$P_n = F_{cr}A_g = 21.82(2.70) = 58.91 \text{ kips}$$

Check width-thickness ratios:

From the dimensions and properties table in the *Manual,* the width-thickness ratio for the larger overall dimension is

$$\frac{h}{t} = 66.0$$

The ratio for the smaller dimension is

$$\frac{b}{t} = 31.5$$

From AISC Table B4.1, case 12 (and Figure 4.9 in this book), the upper limit for nonslender elements is

$$1.40\sqrt{\frac{E}{F_y}} = 1.40\sqrt{\frac{29,000}{46}} = 35.15$$

Since $h/t > 1.40\sqrt{E/F_y}$, the larger dimension element is slender and the local buckling strength must be computed. (Although the limiting width-thickness ratio is labeled *b/t* in the table, that is a generic notation, and it applies to h/t as well.)

Because this cross-sectional element is a stiffened element, $Q_s = 1.0$, and Q_a must be computed from AISC Section E7.2. The shape is a rectangular section of uniform thickness, so AISC E7.2(b) applies, provided that

$$\frac{b}{t} \geq 1.40\sqrt{\frac{E}{f}},$$

where

$$f = \frac{P_n}{A_{eff}}$$

and A_{eff} is the reduced effective area. The Specification user note for square and rectangular sections permits a value of $f = F_y$ to be used in lieu of determining f by iteration. From AISC Equation E7-18, the effective width of the slender element is

$$b_e = 1.92t\sqrt{\frac{E}{f}}\left[1 - \frac{0.38}{b/t}\sqrt{\frac{E}{f}}\right] \leq b \qquad \text{(AISC Equation E7-18)}$$

For the 8-inch side, using $f = F_y$ and the *design* thickness from the dimensions and properties table,

$$b_e = 1.92(0.116)\sqrt{\frac{29{,}000}{46}}\left[1 - \frac{0.38}{(66.0)}\sqrt{\frac{29{,}000}{46}}\right] = 4.784 \text{ in.}$$

From AISC B4.2(d) and the discussion in Part 1 of the *Manual,* the unreduced length of the 8-inch side between the corner radii can be taken as

$$b = 8 - 2(1.5t) = 8 - 2(1.5)(0.116) = 7.652 \text{ in.}$$

where the corner radius is taken as 1.5 times the design thickness.

The total loss in area is therefore

$$2(b - b_e)t = 2(7.652 - 4.784)(0.116) = 0.6654 \text{ in.}^2$$

and the reduced area is

$$A_{eff} = 2.70 - 0.6654 = 2.035 \text{ in.}^2$$

The reduction factor is

$$Q_a = \frac{A_{eff}}{A} = \frac{2.035}{2.70} = 0.7537$$

$$Q = Q_s Q_a = 1.0(0.7537) = 0.7537$$

Compute the local buckling strength.

$$4.71\sqrt{\frac{E}{QF_y}} = 4.71\sqrt{\frac{29{,}000}{0.7537(46)}} = 136.2$$

$$\frac{KL}{r} = 105.3 < 136.2 \qquad \therefore \text{ use AISC Equation E7-2.}$$

$$F_{cr} = Q\left(0.658^{\frac{QF_y}{F_e}}\right)F_y = 0.7537\left(0.658^{\frac{0.7537(46)}{25.81}}\right)46 = 19.76 \text{ ksi}$$

$$P_n = F_{cr}A_g = 19.76(2.70) = 53.35 \text{ kips}$$

Since this is less than the flexural buckling strength of 58.91 kips, local buckling controls.

LRFD Solution Design strength = $\phi_c P_n = 0.90(53.35) = 48.0$ kips

ASD Solution

$$\text{Allowable strength} = \frac{P_n}{\Omega} = \frac{53.35}{1.67} = 32.0 \text{ kips}$$

(Allowable stress = $0.6F_{cr} = 0.6(19.76) = 11.9$ ksi)

Alternative Solution with f determined by Iteration

As an initial trial value use

$f = F_{cr} = 19.76$ ksi (the value obtained above after using an initial value of $f = F_y$)

$$b_e = 1.92(0.116)\sqrt{\frac{29{,}000}{19.76}}\left[1 - \frac{0.38}{(66.0)}\sqrt{\frac{29{,}000}{19.76}}\right] = 6.65 \text{ in.}$$

The total loss in area is

$$2(b - b_e)t = 2(7.652 - 6.65)(0.116) = 0.2325 \text{ in.}^2$$

and the reduced area is

$$A_{eff} = 2.70 - 0.2325 = 2.468 \text{ in.}^2$$

The reduction factor is

$$Q_a = \frac{A_{eff}}{A} = \frac{2.468}{2.70} = 0.9141$$

$$Q = Q_s Q_a = 1.0(0.9141) = 0.9141$$

Compute the local buckling strength.

$$4.71\sqrt{\frac{E}{QF_y}} = 4.71\sqrt{\frac{29{,}000}{0.9141(46)}} = 123.7$$

$$\frac{KL}{r} = 105.3 < 123.7 \qquad \therefore \text{ use AISC Equation E7-2.}$$

$$F_{cr} = Q\left(0.658^{\frac{QF_y}{F_e}}\right)F_y$$

$$= 0.9141\left(0.658^{\frac{0.9141(46)}{25.81}}\right)46 = 21.26 \text{ ksi} \neq 19.76 \text{ ksi (the assumed value)}$$

Try $f = 21.26$ ksi:

$$b_e = 1.92(0.116)\sqrt{\frac{29{,}000}{21.26}}\left[1-\frac{0.38}{(66.0)}\sqrt{\frac{29{,}000}{21.26}}\right] = 6.477 \text{ in.}$$

The total loss in area is

$$2(b-b_e)t = 2(7.652-6.477)(0.116) = 0.2726 \text{ in.}^2$$

and the reduced area is

$$A_{eff} = 2.70 - 0.2726 = 2.427 \text{ in.}^2$$

The reduction factor is

$$Q_a = \frac{A_{eff}}{A} = \frac{2.427}{2.70} = 0.8989$$

$$Q = Q_s Q_a = 1.0(0.8989) = 0.8989$$

Compute the local buckling strength.

$$4.71\sqrt{\frac{E}{QF_y}} = 4.71\sqrt{\frac{29{,}000}{0.8989(46)}} = 124.7$$

$$\frac{KL}{r} = 105.3 < 124.7 \qquad \therefore \text{ use AISC Equation E7-2.}$$

$$F_{cr} = Q\left(0.658^{\frac{QF_y}{F_e}}\right)F_y = 0.8989\left(0.658^{\frac{0.8989(46)}{25.81}}\right)46$$

$$= 21.15 \text{ ksi} \neq 21.26 \text{ ksi}$$

Try $f = 21.15$ ksi:

$$b_e = 1.92(0.116)\sqrt{\frac{29{,}000}{21.15}}\left[1-\frac{0.38}{(66.0)}\sqrt{\frac{29{,}000}{21.15}}\right] = 6.489 \text{ in.}$$

The total loss in area is

$$2(b-b_e)t = 2(7.652-6.489)(0.116) = 0.2698 \text{ in.}^2$$

and the reduced area is

$$A_{eff} = 2.70 - 0.2698 = 2.430 \text{ in.}^2$$

The reduction factor is

$$Q_a = \frac{A_{eff}}{A} = \frac{2.430}{2.70} = 0.9000$$

$$Q = Q_s Q_a = 1.0(0.9000) = 0.9000$$

Compute the local buckling strength.

$$4.71\sqrt{\frac{E}{QF_y}} = 4.71\sqrt{\frac{29,000}{0.9000(46)}} = 124.7$$

$$\frac{KL}{r} = 105.3 < 124.7 \qquad \therefore \text{ use AISC Equation E7-2.}$$

$$F_{cr} = Q\left(0.658^{\frac{QF_y}{F_e}}\right)F_y$$

$$= 0.9000\left(0.658^{\frac{0.9000(46)}{25.81}}\right)46 = 21.16 \text{ ksi} \approx 21.15 \text{ ksi (convergence)}$$

Recall that AISC Equation E7-18 for b_e applies when $b/t \geq 1.40\sqrt{E/f}$. In the present case,

$$1.40\sqrt{\frac{E}{f}} = 1.40\sqrt{\frac{29,000}{21.16}} = 51.8$$

Since 66 > 51.8, AISC Equation E7-18 does apply.

$$P_n = F_{cr}A_g = 21.16(2.70) = 57.13 \text{ kips} \qquad \therefore \text{ local buckling controls.}$$

LRFD Solution Design strength $= \phi_c P_n = 0.90(57.13) = 51.4$ kips

ASD Solution Allowable strength $\dfrac{P_n}{\Omega} = \dfrac{57.13}{1.67} = 34.2$ kips

(Allowable stress $= 0.6F_{cr} = 0.6(21.16) = 12.7$ ksi)

4.5 TABLES FOR COMPRESSION MEMBERS

The *Manual* contains many useful tables for analysis and design. For compression members whose strength is governed by flexural buckling (that is, not local buckling), Table 4-22 in Part 4 of the *Manual,* "Design of Compression Members," can be used. This table gives values of $\phi_c F_{cr}$ (for LRFD) and F_{cr}/Ω_c (for ASD) as a function of KL/r for various values of F_y. This table stops at the recommended upper limit of KL/r = 200. The available strength tables, however, are the most useful. These tables, which we will refer to as the "column load tables," give the available strengths of selected shapes, both $\phi_c P_n$ for LRFD and P_n/Ω_c for ASD, as a function of the effective length KL. These tables include values of KL up to those corresponding to $KL/r = 200$.

The use of the tables is illustrated in the following example.

Example 4.5 Compute the available strength of the compression member of Example 4.2 with the aid of (a) Table 4-22 and (b) the column load tables.

LRFD Solution

a. From Example 4.2, $KL/r = 96.77$ and $F_y = 50$ ksi. Values of $\phi_c F_{cr}$ in Table 4-22 are given only for integer values of KL/r; for decimal values, KL/r may be rounded *up* or linear interpolation may be used. For uniformity, we use interpolation in this book for all tables unless otherwise indicated. For $KL/r = 96.77$ and $F_y = 50$ ksi,

$$\phi_c F_{cr} = 22.67 \text{ ksi}$$

$$\phi_c P_n = \phi_c F_{cr} A_g = 22.67(21.8) = 494 \text{ kips}$$

b. The column load tables in Part 4 of the *Manual* give the available strength for selected W-, HP-, single-angle, WT-, HSS, pipe, double-angle, and composite shapes. (We cover composite construction in Chapter 9.) The tabular values for the symmetrical shapes (W, HP, HSS and pipe) were calculated by using the minimum radius of gyration for each shape. From Example 4.2, $K = 1.0$, so

$$KL = 1.0(20) = 20 \text{ ft}$$

For a W14 × 74, $F_y = 50$ ksi and $KL = 20$ ft,

$$\phi_c P_n = 494 \text{ kips}$$

ASD Solution

a. From Example 4.2, $KL/r = 96.77$ and $F_y = 50$ ksi. By interpolation, for $KL/r = 96.77$ and $F_y = 50$ ksi,

$$F_{cr}/\Omega_c = 15.07 \text{ ksi}$$

Note that this is the allowable stress, $F_a = 0.6F_{cr}$. Therefore, the allowable strength is

$$\frac{P_n}{\Omega_c} = F_a A_g = 15.07(21.8) = 329 \text{ kips}$$

b. From Example 4.2, $K = 1.0$, so

$$KL = 1.0(20) = 20 \text{ ft}$$

From the column load tables, for a W14 × 74 with $F_y = 50$ ksi and $KL = 20$ ft,

$$\frac{P_n}{\Omega_c} = 329 \text{ kips}$$

■

The values from Table 4-22 are based on flexural buckling and AISC Equations E3-2 and E3-3. Thus, local stability is assumed, and width-thickness ratio limits must not be exceeded. Although some shapes in the column load tables exceed those limits (and they are identified with a "c" footnote), the tabulated strength has been computed according to the requirements of AISC Section E7, "Members with Slender Elements," and no further reduction is needed.

From a practical standpoint, if a compression member to be analyzed can be found in the column load tables, then these tables should be used. Otherwise, Table 4-22 can be used for the flexural buckling strength. If the member has slender elements, the local buckling strength must be computed using the provisions of AISC E7.

4.6 DESIGN

The selection of an economical rolled shape to resist a given compressive load is simple with the aid of the column load tables. Enter the table with the effective length and move horizontally until you find the desired available strength (or something slightly larger). In some cases, you must continue the search to be certain that you have found the lightest shape. Usually the category of shape (W, WT, etc.) will have been decided upon in advance. Often the overall nominal dimensions will also be known because of architectural or other requirements. As pointed out earlier, all tabulated values correspond to a slenderness ratio of 200 or less. The tabulated unsymmetrical shapes — the structural tees and the single and double angles — require special consideration and are covered in Section 4.8.

Example 4.6 A compression member is subjected to service loads of 165 kips dead load and 535 kips live load. The member is 26 feet long and pinned at each end. Use A992 steel and select a W14 shape.

LRFD Solution Calculate the factored load:

$$P_u = 1.2D + 1.6L = 1.2(165) + 1.6(535) = 1054 \text{ kips}$$

$$\therefore \text{ Required design strength } \phi_c P_n = 1054 \text{ kips.}$$

From the column load tables for $KL = 1.0(26) = 26$ ft, a W14 × 145 has a design strength of 1230 kips.

Answer Use a W14 × 145.

ASD Solution Calculate the total applied load:

$$P_a = D + L = 165 + 535 = 700 \text{ kips}$$

$$\therefore \text{ Required allowable strength } \frac{P_n}{\Omega_c} = 700 \text{ kips}$$

From the column load tables for $KL = 1.0(26) = 26$ ft, a W14 × 132 has an allowable strength of 702 kips.

Answer Use a W14 × 132.

Example 4.7 Select the lightest W-shape that can resist a service dead load of 62.5 kips and a service live load of 125 kips. The effective length is 24 feet. Use ASTM A992 steel.

Solution The appropriate strategy here is to find the lightest shape for each nominal depth in the column load tables and then choose the lightest overall.

LRFD Solution The factored load is

$$P_u = 1.2D + 1.6L = 1.2(62.5) + 1.6(125) = 275 \text{ kips}$$

From the column load tables, the choices are as follows:

W8: There are no W8s with $\phi_c P_n \geq 275$ kips.

W10: W10 × 54, $\phi_c P_n = 282$ kips

W12: W12 × 58, $\phi_c P_n = 293$ kips

W14: W14 × 61, $\phi_c P_n = 293$ kips

Note that the strength is not proportional to the weight (which is a function of the cross-sectional area).

Answer Use a W10×54

ASD Solution The total applied load is

$$P_a = D + L = 62.5 + 125 = 188 \text{ kips}$$

From the column load tables, the choices are as follows:

W8: There are no W8s with $P_n/\Omega_c \geq 188$ kips.

W10: W10 × 54, $\dfrac{P_n}{\Omega_c} = 188$ kips

W12: W12 × 58, $\dfrac{P_n}{\Omega_c} = 195$ kips

W14: W14 × 61, $\dfrac{P_n}{\Omega_c} = 195$ kips

Note that the strength is not proportional to the weight (which is a function of the cross-sectional area).

Answer Use a W10 × 54. ■

For shapes not in the column load tables, a trial-and-error approach must be used. The general procedure is to assume a shape and then compute its strength. If the strength is too small (unsafe) or too large (uneconomical), another trial must be made. A systematic approach to making the trial selection is as follows:

1. Assume a value for the critical buckling stress F_{cr}. Examination of AISC Equations E3-2 and E3-3 shows that the theoretically maximum value of F_{cr} is the yield stress F_y.
2. Determine the required area. For LRFD,

$$\phi_c F_{cr} A_g \geq P_u$$

$$A_g \geq \frac{P_u}{\phi_c F_{cr}}$$

For ASD,

$$0.6F_{cr} \geq \frac{P_a}{A_g}$$

$$A_g \geq \frac{P_a}{0.6F_{cr}}$$

3. Select a shape that satisfies the area requirement.
4. Compute F_{cr} and the strength for the trial shape.
5. Revise if necessary. If the available strength is very close to the required value, the next tabulated size can be tried. Otherwise, repeat the entire procedure, using the value of F_{cr} found for the current trial shape as a value for Step 1.
6. Check local stability (check width–thickness ratios). Revise if necessary.

Example 4.8 Select a W18 shape of A992 steel that can resist a service dead load of 100 kips and a service live load of 300 kips. The effective length *KL* is 26 feet.

LRFD Solution $P_u = 1.2D + 1.6L = 1.2(100) + 1.6(300) = 600$ kips
Try $F_{cr} = 33$ ksi (an arbitrary choice of two-thirds F_y):

$$\text{Required } A_g = \frac{P_u}{\phi_c F_{cr}} = \frac{600}{0.90(33)} = 20.2 \text{ in.}^2$$

Try a W18 × 71:

$$A_g = 20.8 \text{ in.}^2 > 20.2 \text{ in.}^2 \quad \text{(OK)}$$

$$\frac{KL}{r_{min}} = \frac{26 \times 12}{1.70} = 183.5 < 200 \quad \text{(OK)}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(183.5)^2} = 8.5 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113$$

Since $\frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}}$, AISC Equation E3-3 applies.

$$F_{cr} = 0.877F_e = 0.877(8.5) = 7.455 \text{ ksi}$$
$$\phi_c P_n = \phi_c F_{cr} A_g = 0.90(7.455)(20.8) = 140 \text{ kips} < 600 \text{ kips} \qquad \text{(N.G.)}$$

Because the initial estimate of F_{cr} was so far off, assume a value about halfway between 33 and 7.455 ksi. Try $F_{cr} = 20$ ksi.

$$\text{Required } A_g = \frac{P_u}{\phi_c F_{cr}} = \frac{600}{0.90(20)} = 33.3 \text{ in.}^2$$

Try a W18 × 119:

$$A_g = 35.1 \text{ in.}^2 > 33.3 \text{ in.}^2 \quad \text{(OK)}$$
$$\frac{KL}{r_{\min}} = \frac{26 \times 12}{2.69} = 116.0 < 200 \qquad \text{(OK)}$$
$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(116.0)^2} = 21.27 \text{ ksi}$$

Since $\frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}} = 113$, AISC Equation E3-3 applies.

$$F_{cr} = 0.877F_e = 0.877(21.27) = 18.65 \text{ ksi}$$
$$\phi_c P_n = \phi_c F_{cr} A_g = 0.90(18.65)(35.1) = 589 \text{ kips} < 600 \text{ kips} \qquad \text{(N.G.)}$$

This is very close, so try the next larger size.

Try a W18 × 130:

$$A_g = 38.2 \text{ in.}^2$$
$$\frac{KL}{r_{\min}} = \frac{26 \times 12}{2.70} = 115.6 < 200 \qquad \text{(OK)}$$
$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(115.6)^2} = 21.42 \text{ ksi}$$

Since $\frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}} = 113$, AISC Equation E3-3 applies.

$$F_{cr} = 0.877F_e = 0.877(21.42) = 18.79 \text{ ksi}$$
$$\phi_c P_n = \phi_c F_{cr} A_g = 0.90(18.79)(38.2) = 646 \text{ kips} > 600 \text{ kips} \qquad \text{(OK.)}$$

This shape is not slender (there is no footnote in the dimensions and properties table to indicate that it is), so local buckling does not have to be investigated.

Answer Use a W18 × 130.

ASD Solution The ASD solution procedure is essentially the same as for LRFD, and the same trial values of F_{cr} will be used here.

$$P_a = D + L = 100 + 300 = 400 \text{ kips}$$

Try $F_{cr} = 33$ ksi (an arbitrary choice of two-thirds F_y):

$$\text{Required } A_g = \frac{P_a}{0.6F_{cr}} = \frac{400}{0.6(33)} = 20.2 \text{ in.}^2$$

Try a W18 × 71:

$$A_g = 20.8 \text{ in.}^2 > 20.2 \text{ in.}^2 \qquad \text{(OK)}$$
$$\frac{KL}{r_{\min}} = \frac{26 \times 12}{1.70} = 183.5 < 200 \qquad \text{(OK)}$$
$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(183.5)^2} = 8.5 \text{ ksi}$$
$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113$$

Since $\dfrac{KL}{r} > 4.71\sqrt{\dfrac{E}{F_y}}$, AISC Equation E3-3 applies.

$$F_{cr} = 0.877F_e = 0.877(8.5) = 7.455 \text{ ksi}$$
$$\frac{P_n}{\Omega_c} = 0.6F_{cr}A_g = 0.6(7.455)(20.8) = 93.0 \text{ kips} < 400 \text{ kips} \qquad \text{(N.G.)}$$

Because the initial estimate of F_{cr} was so far off, assume a value about halfway between 33 and 7.455 ksi. Try $F_{cr} = 20$ ksi.

$$\text{Required } A_g = \frac{P_a}{0.6F_{cr}} = \frac{400}{0.6(20)} = 33.3 \text{ in.}^2$$

Try a W18 × 119:

$$A_g = 35.1 \text{ in.}^2 > 33.3 \text{ in.}^2 \qquad \text{(OK)}$$

$$\frac{KL}{r_{\min}} = \frac{26 \times 12}{2.69} = 116.0 < 200 \qquad \text{(OK)}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(116.0)^2} = 21.27 \text{ ksi}$$

Since $\dfrac{KL}{r} > 4.71\sqrt{\dfrac{E}{F_y}} = 113$, AISC Equation E3-3 applies.

$$F_{cr} = 0.877F_e = 0.877(21.27) = 18.65 \text{ ksi}$$

$$0.6F_{cr}A_g = 0.6(18.65)(35.1) = 393 \text{ kips} < 400 \text{ kips} \qquad \text{(N.G.)}$$

This is very close, so try the next larger size.

Try a W18 × 130:

$$A_g = 38.2 \text{ in.}^2$$

$$\frac{KL}{r_{\min}} = \frac{26 \times 12}{2.70} = 115.6 < 200 \qquad \text{(OK)}$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(115.6^2)} = 21.42 \text{ ksi}$$

Since $\dfrac{KL}{r} > 4.71\sqrt{\dfrac{E}{F_y}} = 113$, AISC Equation E3-3 applies.

$$F_{cr} = 0.877F_e = 0.877(21.42) = 18.79 \text{ ksi}$$

$$0.6F_{cr}A_g = 0.6(18.79)(38.2) = 431 \text{ kips } < 400 \text{ kips} \qquad \text{(OK)}$$

This shape is not slender (there is no footnote in the dimensions and properties table to indicate that it is), so local buckling does not have to be investigated.

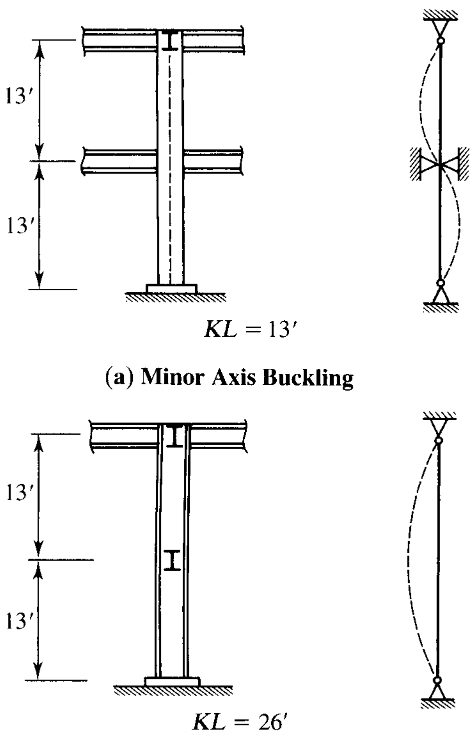
Answer Use a W18 × 130.

4.7 MORE ON EFFECTIVE LENGTH

We introduced the concept of effective length in Section 4.2, "Column Theory." All compression members are treated as pin-ended regardless of the actual end conditions but with an effective length KL that may differ from the actual length. With this modification, the load capacity of compression members is a function of only the slenderness ratio and modulus of elasticity. For a given material, the load capacity is a function of the slenderness ratio only.

If a compression member is supported differently with respect to each of its principal axes, the effective length will be different for the two directions. In Figure 4.10, a W-shape is used as a column and is braced by horizontal members in two perpendicular directions at the top. These members prevent translation of the column in all directions, but the connections, the details of which are not shown, permit small rotations to take place. Under these conditions, the member can be treated as pin-connected at the top. For the same reasons, the connection to the support at the bottom may also be treated as a pin connection. Generally speaking, a rigid, or fixed, condition is very difficult to achieve, and unless some special provisions are made,

FIGURE 4.10



(a) Minor Axis Buckling

(b) Major Axis Buckling

ordinary connections will usually closely approximate a hinge or pin connection. At midheight, the column is braced, but only in one direction.

Again, the connection prevents translation, but no restraint against rotation is furnished. This brace prevents translation perpendicular to the weak axis of the cross section but provides no restraint perpendicular to the strong axis. As shown schematically in Figure 4.10, if the member were to buckle about the major axis, the effective length would be 26 feet, whereas buckling about the minor axis would have to be in the second buckling mode, corresponding to an effective length of 13 feet. Because its strength decreases with increasing KL/r, a column will buckle in the direction corresponding to the largest slenderness ratio, so K_xL/r_x must be compared with K_yL/r_y. In Figure 4.10, the ratio $26(12)/r_x$ must be compared with $13(12)/r_y$ (where r_x and r_y are in inches), and the larger ratio would be used for the determination of the axial compressive strength.

Example 4.9 A W12 × 58, 24 feet long, is pinned at both ends and braced in the weak direction at the third points, as shown in Figure 4.11. A992 steel is used. Determine the available compressive strength.

Solution

$$\frac{K_xL}{r_x} = \frac{24(12)}{5.28} = 54.55$$

$$\frac{K_yL}{r_y} = \frac{8(12)}{2.51} = 38.25$$

K_xL/r_x, the larger value, controls.

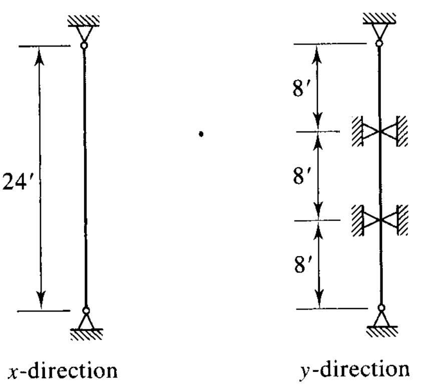
LRFD Solution From Table 4-22 with $KL/r = 54.55$,

$$\phi_c F_{cr} = 36.24 \text{ ksi}$$

$$\phi_c P_n = \phi_c F_{cr} A_g = 36.24(17.0) = 616 \text{ kips}$$

Answer Design strength = 616 kips.

FIGURE 4.11



ASD Solution From Table 4-22 with $KL/r = 54.55$,

$$\frac{F_{cr}}{\Omega_c} = 24.09 \text{ ksi}$$

$$\frac{P_n}{\Omega_c} = \frac{F_{cr}}{\Omega_c} A_g = 24.09(17.0) = 410 \text{ kips}$$

Answer Allowable strength = 410 kips.

The available strengths given in the column load tables are based on the effective length with respect to the y-axis. A procedure for using the tables with $K_x L$, however, can be developed by examining how the tabular values were obtained. Starting with a value of KL, the strength was obtained by a procedure similar to the following.

- KL was divided by r_y to obtain KL/r_y.
- F_{cr} was computed.
- The available strengths, $\phi_c P_n$ for LRFD and P_n/Ω_c for ASD, were computed.

Thus the tabulated strengths are based on the values of KL being equal to $K_y L$. If the capacity with respect to x-axis buckling is desired, the table can be entered with

$$KL = \frac{K_x L}{r_x/r_y}$$

and the tabulated load will be based on

$$\frac{KL}{r_y} = \frac{K_x L/(r_x/r_y)}{r_y} = \frac{K_x L}{r_x}$$

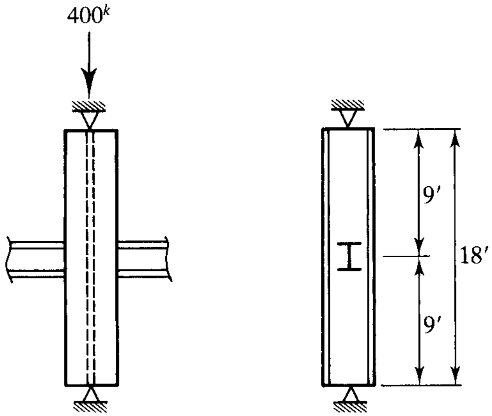
The ratio r_x/r_y is given in the column load tables for each shape listed.

Example 4.10 The compression member shown in Figure 4.12 is pinned at both ends and supported in the weak direction at midheight. A service load of 400 kips, with equal parts of dead and live load, must be supported. Use $F_y = 50$ ksi and select the lightest W-shape.

LRFD Solution Factored load = $P_u = 1.2(200) + 1.6(200) = 560$ kips

Assume that the weak direction controls and enter the column load tables with $KL = 9$ feet. Beginning with the smallest shapes, the first one found that will work is a W8 × 58 with a design strength of 634 kips.

FIGURE 4.12



Check the strong axis:

$$\frac{K_x L}{r_x/r_y} = \frac{18}{1.74} = 10.34 \text{ ft} > 9 \text{ ft}$$

$\therefore K_x L$ controls for this shape.

Enter the tables with $KL = 10.34$ feet. A W8 × 58 has an interpolated strength of

$$\phi_c P_n = 596 \text{ kips} > 560 \text{ kips} \quad \text{(OK)}$$

Next, investigate the W10 shapes. Try a W10 × 49 with a design strength of 569 kips.

Check the strong axis:

$$\frac{K_x L}{r_x/r_y} = \frac{18}{1.71} = 10.53 \text{ ft} > 9 \text{ ft}$$

$\therefore K_x L$ controls for this shape.

Enter the tables with $KL = 10.53$ feet. A W10 × 54 is the lightest W10, with an interpolated design strength of 596 kips.

Continue the search and investigate a W12 × 53 ($\phi_c P_n = 610$ kips for KL = 9 ft):

$$\frac{K_x L}{r_x/r_y} = \frac{18}{2.11} = 8.53 \text{ ft} < 9 \text{ ft}$$

$\therefore K_y L$ controls for this shape, and $\phi_c P_n = 610$ kips.

Determine the lightest W14. The lightest one with a possibility of working is a W14 × 61. It is heavier than the lightest one found so far, so it will not be considered.

Answer Use a W12 × 53.

ASD Solution The required load capacity is $P = 400$ kips. Assume that the weak direction controls and enter the column load tables with $KL = 9$ feet. Beginning with the smallest shapes, the first one found that will work is a W8 × 58 with an allowable strength of 422 kips.

Check the strong axis:

$$\frac{K_x L}{r_x / r_y} = \frac{18}{1.74} = 10.34 \text{ ft} > 9 \text{ ft}$$

$\therefore K_x L$ controls for this shape.

Enter the tables with $KL = 10.34$ feet. A W8 × 58 has an interpolated strength of

$$\frac{P_n}{\Omega_c} = 396 \text{ kips} < 400 \text{ kips} \qquad \text{(N.G.)}$$

The next lightest W8 that will work is a W8 × 67.

$$\frac{K_x L}{r_x / r_y} = \frac{18}{1.75} = 10.29 \text{ ft} > 9 \text{ ft}$$

The interpolated allowable strength is

$$\frac{P_n}{\Omega_c} = 460 \text{ kips} > 400 \text{ kips} \qquad \text{(OK)}$$

Next, investigate the W10 shapes. Try a W10 × 60.

$$\frac{K_x L}{r_x / r_y} = \frac{18}{1.71} = 10.53 \text{ ft} > 9 \text{ ft}$$

The interpolated strength is

$$\frac{P_n}{\Omega_c} = 443 \text{ kips} > 400 \text{ kips} \qquad \text{(OK)}$$

Check the W12 shapes. Try a W12 × 53 ($P_n/\Omega_c = 406$ kips for $KL = 9$ ft):

$$\frac{K_x L}{r_x / r_y} = \frac{18}{2.11} = 8.53 \text{ ft} < 9 \text{ ft}$$

$\therefore K_y L$ controls for this shape, and $P_n/\Omega_c = 406$ kips.

Find the lightest W14. The lightest one with a possibility of working is a W14 ×61. Since it is heavier than the lightest one found so far, it will not be considered.

Answer Use a W12 × 53.

■

Whenever possible, the designer should provide extra support for the weak direction of a column. Otherwise, the member is inefficient: It has an excess of strength in one direction. When K_xL and K_yL are different, K_yL will control unless r_x/r_y is smaller than K_xL/K_yL. When the two ratios are equal, the column has equal strength in both directions. For most of the W-shapes in the column load tables, r_x/r_y ranges between 1.6 and 1.8, but it is as high as 3.1 for some shapes.

Example 4.11

The column shown in Figure 4.13 is subjected to a service dead load of 140 kips and a service live load of 420 kips. Use A992 steel and select a W-shape.

Solution $K_xL = 20$ ft and maximum $K_yL = 8$ ft. The effective length K_xL will control whenever

$$\frac{K_xL}{r_x/r_y} > K_yL$$

or

$$r_x/r_y < \frac{K_xL}{K_yL}$$

In this example,

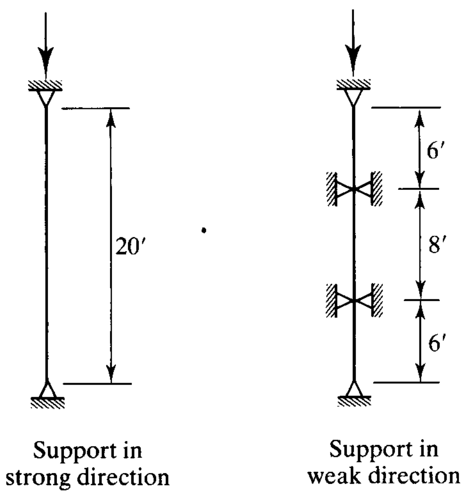
$$\frac{K_xL}{K_yL} = \frac{20}{8} = 2.5$$

so K_xL will control if $r_x/r_y < 2.5$. Since this is true for almost every shape in the column load tables, K_xL probably controls in this example.

Assume $r_x/r_y = 1.7$:

$$\frac{K_xL}{r_x/r_y} = \frac{20}{1.7} = 11.76 > K_yL$$

FIGURE 4.13



LRFD Solution

$$P_u = 1.2D + 1.6L = 1.2(140) + 1.6(420) = 840 \text{ kips}$$

Enter the column load tables with $KL = 12$ feet. There are no W8 shapes with enough load capacity.

Try a W10 × 88 ($\phi_c P_n = 936$ kips):

$$\text{Actual } \frac{K_x L}{r_x/r_y} = \frac{20}{1.73} = 11.56 \text{ ft} < 12 \text{ ft}$$

$$\therefore \phi_c P_n > \text{required 840 kips.}$$

(By interpolation, $\phi_c P_n = 951$ kips.)

Check a W12 × 79:

$$\frac{K_x L}{r_x/r_y} = \frac{20}{1.75} = 11.43 \text{ ft.}$$

$$\phi_c P_n = 900 \text{ kips} > 840 \text{ kips} \qquad \text{(OK)}$$

Investigate W14 shapes. For $r_x/r_y = 2.44$ (the approximate ratio for all likely possibilities),

$$\frac{K_x L}{r_x/r_y} = \frac{20}{2.44} = 8.197 \text{ ft} > K_y L = 8 \text{ ft}$$

For $KL = 9$ ft, a W14 × 74, with a capacity of 853 kips, is the lightest W14-shape. Since 9 feet is a conservative approximation of the actual effective length, this shape is satisfactory.

Answer Use a W14 × 74 (lightest of the three possibilities).

ASD Solution

$$P_a = D + L = 140 + 420 = 560 \text{ kips}$$

Enter the column load tables with $KL = 12$ feet. There are no W8 shapes with enough load capacity. Investigate a W10 × 88 (for $KL = 12$ ft, 12ft, $P_n/\Omega_c = 623$ kips):

$$\text{Actual } \frac{K_x L}{r_x/r_y} = \frac{20}{1.73} = 11.56 \text{ ft} < 12 \text{ ft}$$

$$\therefore \frac{P_n}{\Omega_c} > \text{required 560 kips}$$

(By interpolation, $P_n/\Omega_c = 633$ kips.)

Check a W12 × 79:

$$\frac{K_xL}{r_x/r_y} = \frac{20}{1.75} = 11.43 \text{ ft} < 12 \text{ ft}$$

$$\frac{P_n}{\Omega_c} = 599 \text{ kips} > 560 \text{ kips} \qquad \text{(OK)}$$

Investigate W14 shapes. Try a W14 × 74:

$$\frac{K_xL}{r_x/r_y} = \frac{20}{2.44} = 8.20 > K_yL = 8 \text{ ft}$$

For $KL = 8.20$ ft,

$$\frac{P_n}{\Omega_c} = 581 \text{ kips} > 560 \text{ kips} \qquad \text{(OK)}$$

Answer Use a W14 × 74 (lightest of the three possibilities). ■

For isolated columns that are not part of a continuous frame, Table C-C2.2 in the Commentary to the Specification will usually suffice. Consider, however, the rigid frame in Figure 4.14. The columns in this frame are not independent members but part of a continuous structure. Except for those in the lower story, the columns are restrained at both ends by their connection to beams and other columns. This frame is also unbraced, meaning that horizontal displacements of the frame are possible and all columns are subject to sidesway. If Table C-C2.2 is used for this frame, the lower-story columns are best approximated by condition (f), and a value of $K = 2$ might be used. For a column such as *AB*, a value of $K = 1.2$, corresponding to condition (c), could be selected. A more rational procedure, however, will account for the degree of restraint provided by connecting members.

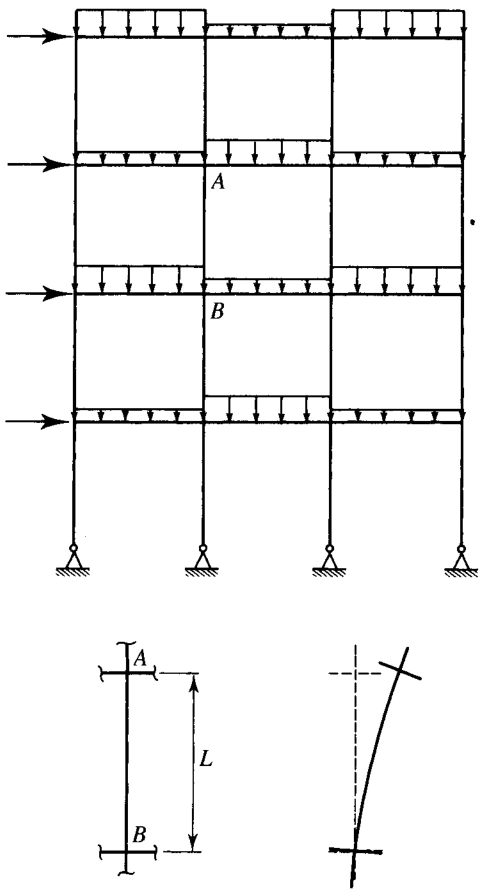
The rotational restraint provided by the beams, or girders, at the end of a column is a function of the rotational stiffnesses of the members intersecting at the joint. The rotational stiffness of a member is proportional to EI/L, where I is the moment of inertia of the cross section with respect to the axis of bending. Gaylord, Gaylord, and Stallmeyer (1992) show that the effective length factor K depends on the ratio of column stiffness to girder stiffness at each end of the member, which can be expressed as

$$G = \frac{\Sigma E_c I_c / L_c}{\Sigma E_g I_g / L_g} = \frac{\Sigma I_c / L_c}{\Sigma I_g / L_g} \qquad (4.12)$$

where

$\Sigma E_c I_c / L_c$ = sum of the stiffnesses of all columns at the end of the column under consideration

FIGURE 4.14



$\Sigma E_g I_g / L_g$ = sum of the stiffnesses of all girders at the end of the column under consideration

$E_c = E_g = E$, the modulus of elasticity of structural steel.

If a very slender column is connected to girders having large cross sections, the girders will effectively prevent rotation of the column. The ends of the column are approximately fixed, and K is relatively small. This condition corresponds to small values of G given by Equation 4.12. However, the ends of stiff columns connected to flexible beams can more freely rotate and approach the pinned condition, giving relatively large values of G and K.

The relationship between G and K has been quantified in the Jackson–Mooreland Alignment Charts (Johnston, 1976), which are reproduced in Figures C-C2.3 and C-C2.4 in the Commentary. To obtain a value of K from one of these nomograms, first calculate the value of G at each end of the column, letting one value be G_A and the other be G_B. Connect G_A and G_B with a straight line, and read the value of K on the middle scale. The effective length factor obtained in this manner is with respect to the axis of bending, which is the axis perpendicular to the plane of the frame. A separate analysis must be made for buckling about the other axis. Normally the beam-to-column connections in this direction will not transmit moment; sidesway is prevented by bracing; and K can be taken as 1.0.

Example 4.12 The rigid frame shown in Figure 4.15 is unbraced. Each member is oriented so that its web is in the plane of the frame. Determine the effective length factor K_x for columns AB and BC.

Solution Column AB:

For joint A,

$$G = \frac{\Sigma I_c/L_c}{\Sigma I_g/L_g} = \frac{833/12 + 1070/12}{1350/20 + 1830/18} = \frac{158.6}{169.2} = 0.94$$

For joint B,

$$G = \frac{\Sigma I_c/L_c}{\Sigma I_g/L_g} = \frac{1070/12 + 1070/15}{169.2} = \frac{160.5}{169.2} = 0.95$$

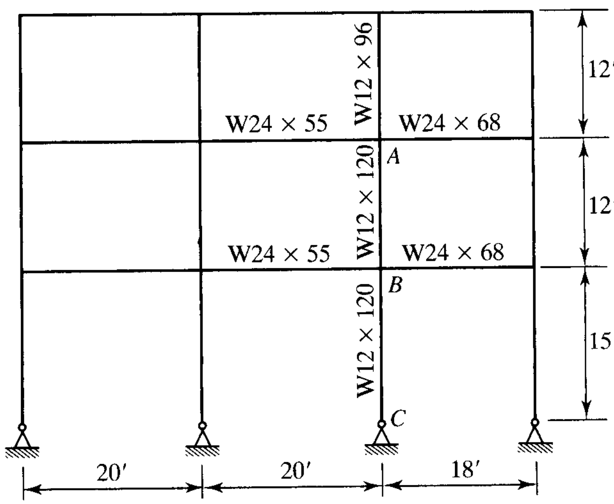
Answer From the alignment chart for sidesway uninhibited (AISC Figure C-C2.4), with $G_A = 0.94$ and $G_B = 0.95$, $K_x = 1.3$ for column AB.

For column BC:

For joint B, as before,

$G = 0.95$

FIGURE 4.15



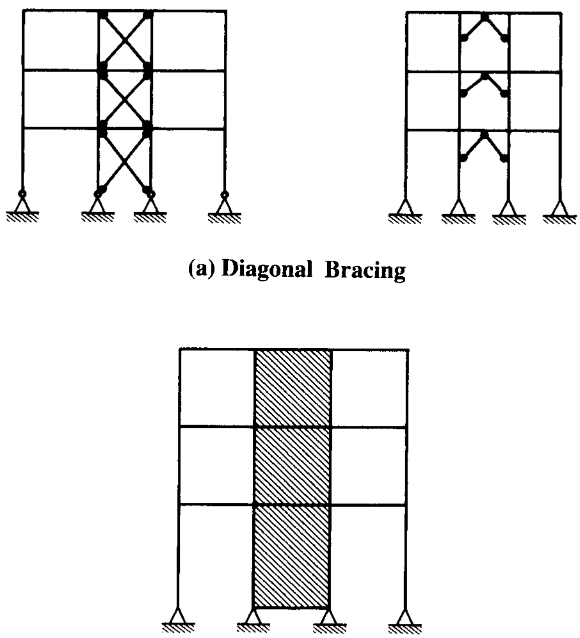
For joint C, a pin connection, the situation is analogous to that of a very stiff column attached to infinitely flexible girders — that is, girders of zero stiffness. The ratio of column stiffness to girder stiffness would therefore be infinite for a perfectly frictionless hinge. This end condition can only be approximated in practice, so the discussion accompanying the alignment chart recommends that G be taken as 10.0.

Answer From the alignment chart with $G_A = 0.95$ and $G_B = 10.0$, $K_x = 1.85$ for column BC.

As pointed out in Example 4.12, for a pinned support, G should be taken as 10.0; for a fixed support, G should be taken as 1.0. The latter support condition corresponds to an infinitely stiff girder and a flexible column, corresponding to a theoretical value of $G = 0$. The discussion accompanying the alignment chart in the Commentary recommends a value of $G = 1.0$ because true fixity will rarely be achieved.

Unbraced frames are able to support lateral loads because of their moment-resisting joints. Often the frame is augmented by a bracing system of some sort; such frames are called *braced frames*. The additional resistance to lateral loads can take the form of diagonal bracing or rigid shear walls, as illustrated in Figure 4.16.

FIGURE 4.16



(a) Diagonal Bracing

(b) Shear Walls
(masonry, reinforced concrete, or steel plate)

In either case, the tendency for columns to sway is blocked within a given panel, or bay, for the full height of the frame. This support system forms a cantilever structure that is resistant to horizontal displacements and also provides horizontal support for the other bays. Depending on the size of the structure, more than one bay may require bracing.

A frame must resist not only the tendency to sway under the action of lateral loads but also the tendency to buckle, or become unstable, under the action of vertical loads. Bracing to stabilize a structure against vertical loading is called *stability bracing*. Appendix 6 of the AISC Specification, "Stability Bracing for Columns and Beams," covers this type of bracing. Two categories are covered: *relative* and *nodal*. With relative bracing, a brace point is restrained relative to adjacent brace points. A relative brace is connected not only to the member to be braced but also to other members, as with diagonal bracing. With relative bracing, both the brace and other members contribute to stabilizing the member to be braced. Nodal bracing provides isolated support at specific locations on the member and is not relative to other brace points or other members. The provisions of AISC Appendix 6 give equations for the required strength and stiffness (resistance to deformation) of stability bracing. The provisions for columns are from the *Guide to Stability Design Criteria* (Galambos, 1998). The required strength and stiffness for stability can be added directly to the requirements for bracing to resist lateral loading. Stability bracing is discussed further in Chapter 5, "Beams," and Chapter 6, "Beam–Columns."

Columns that are members of braced rigid frames are prevented from sidesway and have some degree of rotational restraint at their ends. Thus they are in a category that lies somewhere between cases (a) and (d) in Table C-C2.2 of the Commentary, and K is between 0.5 and 1.0. A value of 1.0 is therefore always conservative for members of braced frames and is the value prescribed by AISC C1.3a unless an analysis is made. Such an analysis can be made with the alignment chart for braced frames. Use of this nomogram would result in an effective length factor somewhat less than 1.0, and some savings could be realized.*

As with any design aid, the alignment charts should be used only under the conditions for which they were derived. These conditions are discussed in Section C2 of the Commentary to the Specification and are not enumerated here. Most of the conditions will usually be approximately satisfied; if they are not, the deviation will be on the conservative side. One condition that usually is not satisfied is the requirement that all behavior be elastic. If the slenderness ratio KL/r is less than $4.71\sqrt{E/F_y}$, the column will buckle inelastically, and the effective length factor obtained from the alignment chart will be overly conservative. A large number of columns are in this category. A convenient procedure for determining K for inelastic columns allows the alignment charts to be used (Yura, 1971; Disque, 1973). To demonstrate the procedure,

*If a frame is braced against sidesway, the beam-to-column connections need not be moment-resisting, and the bracing system could be designed to resist all sidesway tendency. If the connections are not moment-resisting, however, there will be no continuity between columns and girders, and the alignment chart cannot be used. For this type of braced frame, K_x should be taken as 1.0.

we begin with the critical buckling load for an inelastic column given by Equation 4.6b. Dividing it by the cross-sectional area gives the buckling stress:

$$F_{cr} = \frac{\pi^2 E_t}{(KL/r)^2}$$

The rotational stiffness of a column in this state would be proportional to $E_t I_c / L_c$, and the appropriate value of G for use in the alignment chart is

$$G_{\text{inelastic}} = \frac{\Sigma E_t I_c / L_c}{\Sigma E I_g / L_g} = \frac{E_t}{E} G_{\text{elastic}}$$

Because E_t is less than E, $G_{\text{inelastic}}$ is less than G_{elastic}, and the effective length factor K will be reduced, resulting in a more economical design. To evaluate E_t/E, called the *stiffness reduction factor* (denoted by τ_a), consider the following relationship for a column with pinned ends:

$$\frac{F_{cr(\text{inelastic})}}{F_{cr(\text{elastic})}} = \frac{\pi^2 E_t/(L/r)^2}{\pi^2 E/(L/r)^2} = \frac{E_t}{E} \qquad (4.13)$$

AISC uses an approximation for the inelastic portion of the column strength curve, so Equation 4.13 is an approximation when AISC Equations E3-2 and E3-3 are used for F_{cr}.

We can approximate F_{cr} by the compressive strength:

$$F_{cr} = \frac{P_u}{\phi_c A_g} \quad \text{for LRFD}$$

$$= \frac{\Omega_c P_a}{A_g} \quad \text{for ASD}$$

Then in the elastic range, $F_{cr(\text{inelastic})}$ is approximately

$$\frac{P_u}{\phi_c A_g} = 0.658^{(F_y/F_e)} F_y \quad \text{for LRFD}$$

and

$$\frac{\Omega_c P_a}{A_g} = 0.658^{(F_y/F_e)} F_y \quad \text{for ASD}$$

We can solve for F_e, then compute

$$F_{cr(\text{elastic})} = 0.877 F_e$$

The stiffness reduction factor τ_a can then be computed. ■

Example 4.13 Compute the stiffness reduction factor τ_a for an axial compressive stress of 25 ksi and $F_y = 50$ ksi.

LRFD Solution

$$\frac{P_u}{A_g} = 25 \text{ ksi}$$

$$F_{cr(\text{inelastic})} = \frac{P_u}{\phi_c A_g} = \frac{25}{0.90} = 27.78 \text{ ksi} = 0.658^{(F_y/F_e)} F_y$$

or

$$27.78 = 0.658^{(50/F_e)}(50), \qquad F_e = 35.61 \text{ ksi}$$

$$F_{cr(\text{inelastic})} = 0.877 F_e = 0.877(35.61) = 31.23 \text{ ksi}$$

The stiffness reduction factor is therefore

Answer

$$\tau_a = \frac{F_{cr(\text{inelastic})}}{F_{cr(\text{elastic})}} = \frac{27.78}{31.23} = 0.890$$

ASD Solution

$$F_{cr(\text{inelastic})} = \frac{\Omega_c P_a}{A_g} = 1.67(25) = 41.75 \text{ ksi}$$

$$41.75 = 0.658^{(50/F_e)}(50), \qquad F_e = 116.1 \text{ ksi}$$

$$F_{cr(\text{elastic})} = 0.877 F_e = 0.877(116.1) = 101.8 \text{ ksi}$$

Answer

$$\tau_a = \frac{F_{cr(\text{inelastic})}}{F_{cr(\text{elastic})}} = \frac{41.75}{101.8} = 0.410 \text{ ksi}$$

■

Values of the stiffness reduction factor τ_a as a function of P_u/A_g and P_a/A_g are given in Table 4-21 in Part 4 of the *Manual.*

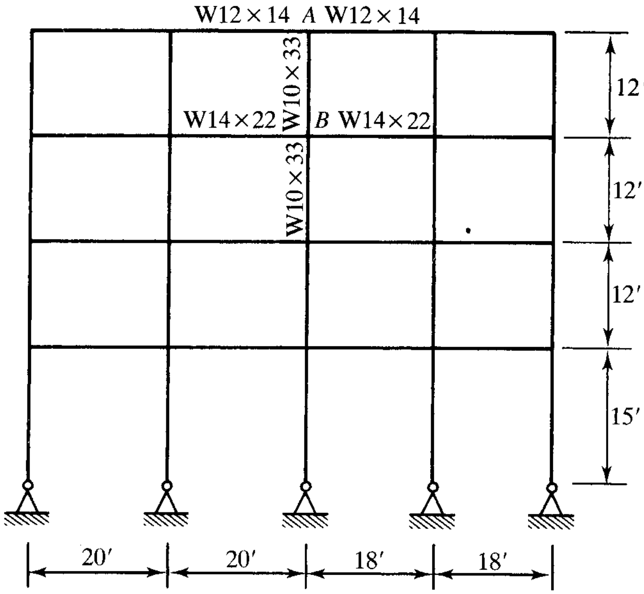
Example 4.14 A rigid unbraced frame is shown in Figure 4.17. All members are oriented so that bending is about the strong axis. Lateral support is provided at each joint by simply connected bracing in the direction perpendicular to the frame. Determine the effective length factors with respect to each axis for member *AB*. The service dead load is 35.5 kips, and the service live load is 142 kips. A992 steel is used.

Solution Compute elastic *G* factors:

For joint *A*,

$$\frac{\Sigma(I_c/L_c)}{\Sigma(I_g/L_g)} = \frac{171/12}{88.6/20 + 88.6/18} = \frac{14.25}{9.352} = 1.52$$

FIGURE 4.17



For joint *B*,

$$\frac{\Sigma(I_c/L_c)}{\Sigma(I_g/L_g)} = \frac{2(171/12)}{199/20 + 199/18} = \frac{28.5}{21.01} = 1.36$$

From the alignment chart for unbraced frames, $K_x = 1.45$, based on elastic behavior. Determine whether the column behavior is elastic or inelastic.

$$\frac{K_x L}{r_x} = \frac{1.45(12 \times 12)}{4.19} = 49.83$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113$$

Since

$$\frac{K_x L}{r_x} < 4.71\sqrt{\frac{E}{F_y}}$$

behavior is inelastic, and the inelastic *K* factor can be used.

LRFD Solution The factored load is

$$P_u = 1.2D + 1.6L = 1.2(35.5) + 1.6(142) = 269.8 \text{ kips}$$

Enter Table 4-21 in Part 4 of the *Manual* with

$$\frac{P_u}{A_g} = \frac{269.8}{9.71} = 27.79 \text{ ksi}$$

and obtain the stiffness reduction factor $\tau_a = 0.8105$ by interpolation.

For joint *A*,

$$G_{\text{inelastic}} = \tau_a \times G_{\text{elastic}} = 0.8105(1.52) = 1.23$$

For joint *B*,

$$G_{\text{inelastic}} = 0.8105(1.36) = 1.10$$

Answer From the alignment chart, $K_x = 1.35$. Because of the support conditions normal to the frame, K_y can be taken as 1.0.

ASD Solution The applied load is

$$P_a = D + L = 35.5 + 142 = 177.5 \text{ kips}$$

Enter Table 4-21 in Part 4 of the *Manual* with

$$\frac{P_a}{A_g} = \frac{177.5}{9.71} = 18.28 \text{ ksi}$$

and obtain the stiffness reduction factor $\tau_a = 0.8198$ by interpolation.
For joint *A*,

$$G_{\text{inelastic}} = \tau_a \times G_{\text{elastic}} = 0.8198(1.52) = 1.25$$

For joint *B*,

$$G_{\text{inelastic}} = 0.8198(1.36) = 1.12$$

Answer From the alignment chart, $K_x = 1.35$. Because of the support conditions normal to the frame, K_y can be taken as 1.0. ■

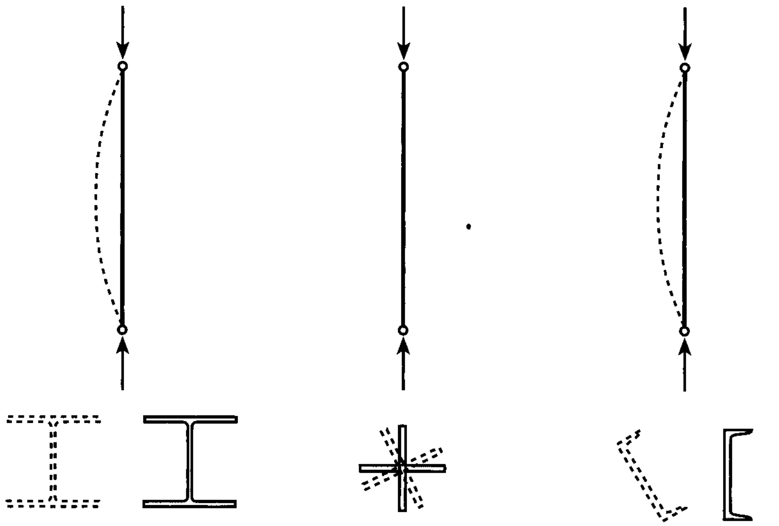
If the end of a column is fixed ($G = 1.0$) or pinned ($G = 10.0$), the value of G at that end should *not* be multiplied by the stiffness reduction factor.

4.8 TORSIONAL AND FLEXURAL-TORSIONAL BUCKLING

When an axially loaded compression member becomes unstable overall (that is, not locally unstable), it can buckle in one of three ways, as shown in Figure 4.18).

1. **Flexural buckling.** We have considered this type of buckling up to now. It is a deflection caused by bending, or flexure, about the axis corresponding to the largest slenderness ratio (Figure 4.18a). This is usually the minor principal axis — the one with the smallest radius of gyration. Compression members with any type of cross-sectional configuration can fail in this way.
2. **Torsional buckling.** This type of failure is caused by twisting about the longitudinal axis of the member. It can occur only with doubly symmetrical cross

FIGURE 4.18



(a) **Flexural Buckling**

(b) **Torsional Buckling** (cruciform shape shown)

(c) **Flexural-torsional Buckling**

sections with very slender cross-sectional elements (Figure 4.18b). Standard hot-rolled shapes are not susceptible to torsional buckling, but members built up from thin plate elements may be and should be investigated. The cruciform shape shown is particularly vulnerable to this type of buckling. This shape can be fabricated from plates as shown in the figure, or built up from four angles placed back to back.

3. **Flexural-torsional buckling.** This type of failure is caused by a combination of flexural buckling and torsional buckling. The member bends and twists simultaneously (Figure 4.18c). This type of failure can occur only with unsymmetrical cross sections, both those with one axis of symmetry — such as channels, structural tees, double-angle shapes, and equal-leg single angles — and those with no axis of symmetry, such as unequal-leg single angles.

The AISC Specification requires an analysis of torsional or flexural-torsional buckling when appropriate. Section E4(a) of the Specification covers double-angle and tee-shaped members, and Section E4(b) provides a more general approach that can be used for any shape. We discuss the general approach first. It is based on first determining a value of F_e, which is analogous to the Euler buckling stress. This stress can then be used with the flexural buckling equations, AISC Equations E3-2 and E3-3. The stress F_e can be defined as the elastic buckling stress corresponding to the controlling mode of failure, whether flexural, torsional, or flexural-torsional.

The equations for F_e given in AISC E4(b)are based on well-established theory given in *Theory of Elastic Stability* (Timoshenko and Gere, 1961). Except for some changes in notation, they are the same equations as those given in that work, with no simplifications. For doubly symmetrical shapes (torsional buckling),

$$F_e = \left[\frac{\pi^2 EC_w}{(K_z L)^2} + GJ\right]\frac{1}{I_x + I_y} \qquad \text{(AISC Equation E4-4)}$$

For singly symmetrical shapes (flexural-torsional buckling),

$$F_e = \frac{F_{ey} + F_{ez}}{2H}\left(1 - \sqrt{1 - \frac{4F_{ey}F_{ez}H}{(F_{ey} + F_{ez})^2}}\right) \qquad \text{(AISC Equation E4-5)}$$

where y is the axis of symmetry.

For shapes with *no* axis of symmetry (flexural-torsional buckling),

$$(F_e - F_{ex})(F_e - F_{ey})(F_e - F_{ez}) - F_e^2(F_e - F_{ey})\left(\frac{x_0}{\bar{r}_0}\right)^2$$

$$- F_e^2(F_e - F_{ex})\left(\frac{y_0}{\bar{r}_0}\right)^2 = 0 \qquad \text{(AISC Equation E4-6)}$$

This last equation is a cubic; F_e is the smallest root. Fortunately, there will be little need for solving this equation, because completely unsymmetrical shapes are rarely used as compression members.

In the above equations, the z-axis is the longitudinal axis. The previously undefined terms in these three equations are defined as

C_w = warping constant (in.^{6})
K_z = effective length factor for *torsional* buckling, which is based on the amount of end restraint against twisting about the longitudinal axis
G = shear modulus (ksi) = 11,200 ksi for structural steel
J = torsional constant (equal to the polar moment of inertia only for circular cross sections) (in.^{4})

$$F_{ex} = \frac{\pi^2 E}{(K_x L/r_x)^2} \qquad \text{(AISC Equation E4-9)}$$

$$F_{ey} = \frac{\pi^2 E}{(K_y L/r_y)^2} \qquad \text{(AISC Equation E4-10)}$$

where y is the axis of symmetry for singly symmetrical shapes.

$$F_{ez} = \left(\frac{\pi^2 EC_w}{(K_z L)^2} + GJ\right)\frac{1}{A_g \bar{r}_0^2} \qquad \text{(AISC Equation E4-11)}$$

$$H = 1 - \frac{x_0^2 + y_0^2}{\bar{r}_0^2} \qquad \text{(AISC Equation E4-8)}$$

TABLE 4.1

Shape	Constants
W, M, S, HP, WT, MT, ST	J, C_w (In addition, the Manual Companion CD gives values of $\bar{r}_0$, and H for WT, MT, and ST shapes)
C	J, C_w, $\bar{r}_0$, H
MC, Angles	J, C_w $\bar{r}_0$, (In addition, the Manual Companion CD gives values of H for MC and angle shapes.)
Double Angles	$\bar{r}_0$, H (J and C_w are double the values given for single angles.)

where z is the longitudinal axis and x_0, y_0 are the coordinates of the shear center of the cross section with respect to the centroid (in inches). The shear center is the point on the cross section through which a transverse load on a beam must pass if the member is to bend without twisting.

$$\bar{r}_0^2 = x_0^2 + y_0^2 + \frac{I_x + I_y}{A_g} \qquad \text{(AISC Equation E4-7)}$$

Values of the constants used in the equations for F_e can be found in the dimensions and properties tables in Part 1 of the *Manual.* Table 4.1 shows which constants are given for various types of shapes. Table 4.1 shows that the *Manual* does not give the constants $\bar{r}_0$ and H for tees, although they are given on the Companion CD. They are easily computed, however, if x_0 and y_0 are known. Since x_0 and y_0 are the coordinates of the shear center with respect to the centroid of the cross section, the location of the shear center must be known. For a tee shape, it is located at the intersection of the centerlines of the flange and the stem. Example 4.15 illustrates the computation of $\bar{r}_0$ and H.

As previously pointed out, the need for a torsional buckling analysis of a doubly symmetrical shape will be rare. Similarly, shapes with no axis of symmetry are rarely used for compression members, and flexural-torsional buckling analysis of these types of members will seldom, if ever, need to be done. For these reasons, we limit further consideration to flexural-torsional buckling of shapes with one axis of symmetry. Furthermore, the most commonly used of these shapes, the double angle, is a built-up shape, and we postpone consideration of it until Section 4.9.

For singly symmetrical shapes, the flexural-torsional buckling stress F_e is found from AISC Equation E4-5. In this equation, y is defined as the axis of symmetry (regardless of the orientation of the member), and flexural-torsional buckling will take place only about this axis (flexural buckling about this axis will not occur). The x-axis is subject only to flexural buckling. Therefore, for singly symmetrical shapes, there are two possibilities for the strength: either flexural-torsional buckling about the y-axis (the axis of symmetry) or flexural buckling about the x-axis (Timoshenko and Gere, 1961 and Zahn and Iwankiw, 1989). To determine which one controls, compute the strength corresponding to each axis and use the smaller value.

Example 4.15 Compute the compressive strength of a WT12 × 81 of A992 steel. The effective length with respect to the x-axis is 25 feet 6 inches, the effective length with respect to the y-axis is 20 feet, and the effective length with respect to the z-axis is 20 feet. Use the general approach of AISC E4(b).

Solution Compute the flexural buckling strength for the x-axis:

$$\frac{K_xL}{r_x} = \frac{25.5 \times 12}{3.50} = 87.43$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(87.43)^2} = 37.44 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113$$

Since $\frac{KL}{r} < 4.71\sqrt{\frac{E}{F_y}}$, AISC Equation E3-2 applies.

$$F_{cr} = 0.658^{(F_y/F_e)} F_y = 0.658^{(50/37.44)}(50) = 28.59 \text{ ksi}$$

The nominal strength is

$$P_n = F_{cr}A_g = 28.59(23.9) = 683.3 \text{ kips}$$

Compute the flexural-torsional buckling strength about the y-axis (the axis of symmetry):

$$\frac{K_yL}{r_y} = \frac{20 \times 12}{3.05} = 78.69$$

$$F_{ey} = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(78.69)^2} = 46.22 \text{ ksi}$$

Because the shear center of a tee is located at the intersection of the centerlines of the flange and the stem,

$$x_0 = 0$$

$$y_0 = \bar{y} - \frac{t_f}{2} = 2.70 - \frac{1.22}{2} = 2.090 \text{ in.}$$

$$\bar{r}_0^2 = x_0^2 + y_0^2 + \frac{I_x + I_y}{A_g} = 0 + (2.090)^2 + \frac{293 + 221}{23.9} = 25.87 \text{ in.}^2$$

$$F_{ez} = \left[\frac{\pi^2 EC_w}{(K_zL)^2} + GJ\right]\frac{1}{A_g\bar{r}_0^2}$$

$$= \left[\frac{\pi^2(29{,}000)(43.8)}{(20 \times 12)^2} + 11{,}200(9.22)\right]\frac{1}{23.9(25.87)} = 167.4 \text{ ksi}$$

$$F_{ey} + F_{ez} = 46.22 + 167.4 = 213.6 \text{ ksi}$$

$$H = 1 - \frac{x_0^2 + y_0^2}{\bar{r}_0^2} = 1 - \frac{0 + (2.090)^2}{25.87} = 0.8312$$

(Note that, for tees, the values of $\bar{r}_0$ and H can be found on the manual companion CD.)

$$F_e = \left(\frac{F_{ey} + F_{ez}}{2H}\right)\left[1 - \sqrt{1 - \frac{4F_{ey}F_{ez}H}{(F_{ey} + F_{ez})^2}}\right]$$

$$= \frac{213.6}{2(0.8312)}\left[1 - \sqrt{1 - \frac{4(46.22)(167.4)(0.8312)}{(213.6)^2}}\right] = 43.63 \text{ ksi}$$

To determine which compressive strength equation to use, compare this value of F_e with

$$0.44F_y = 0.44(50) = 22.0 \text{ ksi}$$

Since 43.63 ksi > 22.0 ksi, use AISC Equation E3-2.

$$F_{cr} = 0.658^{(F_y/F_e)} F_y = 0.658^{(50/37.44)}(50) = 28.59 \text{ ksi}$$

The nominal strength is

$$P_n = F_{cr}A_g = 30.95(23.9) = 739.7 \text{ kips}$$

The flexural buckling strength controls, and the nominal strength is 683.3 kips.

Answer For LRFD, the design strength is $\phi_c P_n = 0.90(683.3) = 615$ kips.

For ASD, the allowable stress is $F_a = 0.6F_{cr} = 0.6(28.59) = 17.15$ ksi and the allowable strength is $F_a A_g = 17.15(23.9) = 410$ kips. ■

The procedure for flexural-torsional buckling analysis of double angles and tees given in AISC Section E4(a) is a modification of the procedure given in AISC E4(b). There is also some notational change: F_e becomes F_{cr}, F_{ey} becomes F_{cry}, and F_{ez} becomes F_{crz}.

To obtain F_{crz}, we can drop the first term of AISC Equation E4-11 to get

$$F_{crz} = \frac{GJ}{A_g \bar{r}_0^2} \qquad \text{(AISC Equation E4-3)}$$

This approximation is acceptable because for double angles and tees, the first term is negligible compared to the second term.

The flexural buckling stress F_{cry} is computed with the usual equations of AISC E3, using KL/r corresponding to the y-axis (the axis of symmetry).

The nominal strength can then be computed as

$$P_n = F_{cr}A_g \qquad \text{(AISC Equation E3-1)}$$

where

$$F_{cr} = \left(\frac{F_{cry} + F_{crz}}{2H}\right)\left[1 - \sqrt{1 - \frac{4F_{cry}F_{crz}H}{(F_{cry} + F_{crz})^2}}\right] \qquad \text{(AISC Equation E4-2)}$$

All other terms from Section E4(b) remain unchanged. This procedure, to be used with double angles and tees only, is more accurate than the procedure given in E4(b).

Example 4.16 Compute the strength of the shape in Example 4.15 by using the equations of AISC E4(a).

Solution From Example 4.15, the nominal flexural buckling strength for the x-axis is 683.3 kips (with $F_{cr} = 28.59$ ksi). The following values were also computed in Example 4.15:

$$K_y L/r_y = 78.69$$

$$\bar{r}_0^2 = 25.87 \text{ in.}^2$$

$$H = 0.8312$$

Compute F_{cry} using AISC E3. From AISC Equation E3-4,

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2 E}{(K_y L/r_y)^2} = \frac{\pi^2(29{,}000)}{(78.69)^2} = 46.22 \text{ ksi}$$

Since $K_y L/r_y < 4.71\sqrt{\dfrac{E}{F_y}} = 113$

$$F_{cry} = 0.658^{(F_y/F_e)} F_y = 0.658^{(50/46.22)}(50) = 31.79 \text{ ksi}$$

$$F_{crz} = \frac{GJ}{A_g \bar{r}_0^2} = \frac{11{,}200(9.22)}{23.9(25.87)} = 167.0 \text{ ksi}$$

$$F_{cry} + F_{crz} = 31.79 + 167.0 = 198.8 \text{ ksi}$$

$$F_{cr} = \left(\frac{F_{cry} + F_{crz}}{2H}\right)\left[1 - \sqrt{1 - \frac{4F_{cry}F_{crz}H}{(F_{cry} + F_{crz})^2}}\right]$$

$$= \frac{198.8}{2(0.8312)}\left[1 - \sqrt{1 - \frac{4(31.79)(167.0)(0.8312)}{(198.8)^2}}\right] = 30.63 \text{ ksi}$$

$$P_n = F_{cr} A_g = 30.63(23.9) = 732.1 \text{ kips}$$

The flexural buckling strength controls, and the nominal strength is 683.3 kips.

Answer For LRFD, the design strength is $\phi_c P_n = 0.90(683.3) = 615$ kips.

For ASD, the allowable stress is $F_a = 0.6F_{cr} = 0.6(28.59) = 17.15$ ksi, and the allowable strength is $F_a A_g = 17.15(23.9) = 410$ kips.

The flexural-torsional buckling results of Examples 4.15 and 4.16 show that the error in using the general approach of Section E4(b) for this shape is on the unconservative side. The procedure used in Example 4.16, which is based on AISC Specification E4(a), should always be used for double angles and tees. In practice, however, the strength of most double angles and tees can be found in the column load tables. These tables give two sets of values of the available strength, one based on flexural buckling about the x-axis and one based on flexural-torsional buckling about the y axis. The flexural-torsional buckling strengths are based on the recommended procedure of AISC E4(a).

Available compressive strength tables are also provided for single-angle members. The values of strength in these tables are not based on flexural-torsional buckling theory, but on the provisions of AISC E5.

When using the column load tables for unsymmetrical shapes, there is no need to account for slender compression elements, because that has already been done. If an analysis is being done for a member not in the column load tables, then any element slenderness must be accounted for.

4.9 BUILT-UP MEMBERS

If the cross-sectional properties of a built-up compression member are known, its analysis is the same as for any other compression member, provided the component parts of the cross section are properly connected. AISC E6 contains many details related to this connection, with separate requirements for members composed of two or more rolled shapes and for members composed of plates or a combination of plates and shapes. Before considering the connection problem, we will review the computation of cross-sectional properties of built-up shapes.

The design strength of a built-up compression member is a function of the slenderness ratio KL/r. Hence the principal axes and the corresponding radii of gyration about these axes must be determined. For homogeneous cross sections, the principal axes coincide with the centroidal axes. The procedure is illustrated in Example 4.17. The components of the cross section are assumed to be properly connected.

Example 4.17 The column shown in Figure 4.19 is fabricated by welding a ⅜-inch by 4-inch cover plate to the flange of a W18 × 65. Steel with $F_y = 50$ ksi is used for both components. The effective length is 15 feet with respect to both axes. Assume that the components are connected in such a way that the member is fully effective and compute the strength based on flexural buckling.

FIGURE 4.19

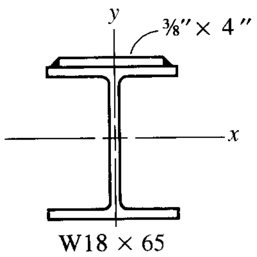


TABLE 4.2

Component	A	y	Ay
Plate	1.500	0.1875	0.2813
W	19.10	9.575	182.9
Σ	20.60		183.2

Solution With the addition of the cover plate, the shape is slightly unsymmetrical, but the flexural-torsional effects will be negligible.

The vertical axis of symmetry is one of the principal axes, and its location need not be computed. The horizontal principal axis will be found by application of the *principle of moments:* The sum of moments of component areas about any axis (in this example, a horizontal axis along the top of the plate will be used) must equal the moment of the total area. We use Table 4.2 to keep track of the computations.

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{183.2}{20.60} = 8.893 \text{ in.}$$

With the location of the horizontal centroidal axis known, the moment of inertia with respect to this axis can be found by using the *parallel-axis theorem:*

$$I = \bar{I} + Ad^2$$

where

$\bar{I}$ = moment of inertia about the centroidal axis of a component area
A = area of the component
I = moment of inertia about an axis parallel to the centroidal axis of the component area
d = perpendicular distance between the two axes

The contributions from each component area are computed and summed to obtain the moment of inertia of the composite area. These computations are shown in Table 4.3, which is an expanded version of Table 4.2. The moment of inertia about the x-axis is

$$I_x = 1193 \text{ in.}^4$$

TABLE 4.3

Component	A	y	Ay	$\bar{I}$	d	$\bar{I} + Ad^2$
Plate	1.500	0.1875	0.2813	0.01758	8.706	113.7
W	19.10	9.575	182.9	1070	0.6820	1079
Σ	20.60		183.2			1193

For the vertical axis,

$$I_y = \frac{1}{12}\left(\frac{3}{8}\right)(4)^3 + 54.8 = 56.80 \text{ in.}^4$$

Since $I_y < I_x$, the y-axis controls.

$$r_{\min} = r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{56.80}{20.60}} = 1.661 \text{ in.}$$

$$\frac{KL}{r_{\min}} = \frac{15 \times 12}{1.661} = 108.4$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2 (29{,}000)}{(108.4)^2} = 24.36 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113$$

Since $\frac{KL}{r} < 4.71\sqrt{\frac{E}{F_y}}$, use AISC Equation E3-2.

$$F_{cr} = 0.658^{(F_y/F_e)} F_y = 0.658^{(50/24.36)}(50) = 21.18 \text{ ksi}$$

The nominal strength is

$$P_n = F_{cr} A_g = 21.18(20.60) = 436.3 \text{ kips}$$

LRFD Solution The design strength is

$$\phi_c P_n = 0.90(436.3) = 393 \text{ kips}$$

ASD Solution From Equation 4.7, the allowable stress is

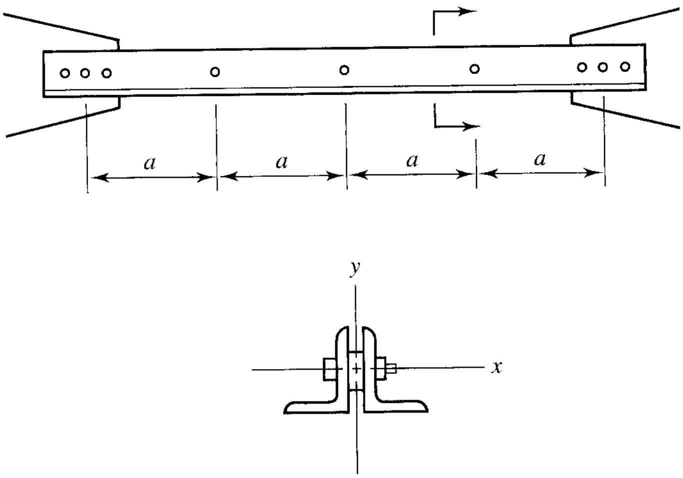
$$F_a = 0.6F_{cr} = 0.6(21.18) = 12.71 \text{ ksi}$$

The allowable strength is

$$F_a A_g = 12.71(20.60) = 262 \text{ kips}$$

Answer Design compressive strength = 393 kips. Allowable compressive strength = 262 kips.

FIGURE 4.20



Connection Requirements for Built-Up Members Composed of Rolled Shapes

The most common built-up shape is one that is composed of rolled shapes, namely, the double-angle shape. This type of member will be used to illustrate the requirements for this category of built-up members. Figure 4.20 shows a truss compression member connected to gusset plates at each end. To maintain the back-to-back separation of the angles along the length, fillers (spacers) of the same thickness as the gusset plate are placed between the angles at equal intervals. The intervals must be small enough that the member functions as a unit. If the member buckles about the x-axis (flexural buckling), the connectors are not subjected to any calculated load, and the connection problem is simply one of maintaining the relative positions of the two components. To ensure that the built-up member acts as a unit, AISC E6.2 requires that the slenderness of an individual component be no greater than three-fourths of the slenderness of the built-up member; that is,

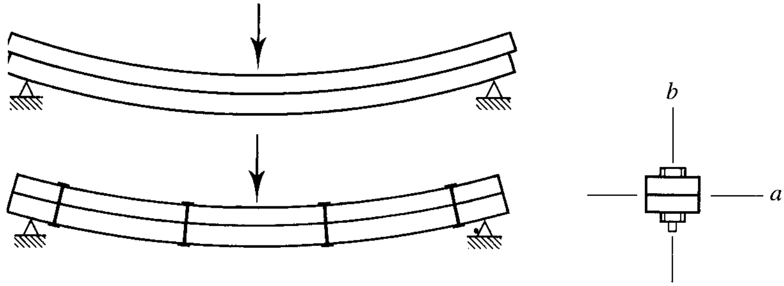
$$\frac{Ka}{r_i} \leq \frac{3}{4}\frac{KL}{r} \tag{4.14}$$

where

a = spacing of the connectors
r_i = smallest radius of gyration of the component
Ka/r_i = effective slenderness ratio of the component
KL/r = maximum slenderness ratio of the built-up member

If the member buckles about the axis of symmetry — that is, if it is subjected to flexural-torsional buckling about the y-axis — the connectors are subjected to shearing forces. This condition can be visualized by considering two planks used as a beam, as shown in Figure 4.21. If the planks are unconnected, they will slip along the surface of contact when loaded and will function as two separate beams. When connected by bolts (or any other fasteners, such as nails), the two planks will behave as a

FIGURE 4.21



unit, and the resistance to slip will be provided by shear in the bolts. This behavior takes place in the double-angle shape when bending about its y-axis. If the plank beam is oriented so that bending takes place about its other axis (the b-axis), then both planks bend in exactly the same manner, and there is no slippage and hence no shear. This behavior is analogous to bending about the x-axis of the double-angle shape. When the fasteners are subjected to shear, a modified slenderness ratio larger than the actual value may be required.

AISC E6 considers two categories of connectors: (1) snug-tight bolts and (2) welds or fully-tensioned bolts. We cover these connection methods in detail in Chapter 7, "Simple Connections." The column load tables for double angles are based on the use of welds or fully tightened bolts. For this category,

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_0^2 + 0.82\frac{\alpha^2}{(1+\alpha^2)}\left(\frac{a}{r_{ib}}\right)^2} \qquad \text{(AISC Equation E6-2)}$$

where

$(KL/r)_m$ = modified slenderness ratio
$(KL/r)_0$ = original unmodified slenderness ratio
r_{ib} = radius of gyration of component about axis parallel to member axis of buckling
α = separation ratio = $h/2r_{ib}$
h = distance between component centroids (perpendicular to member axis of buckling)

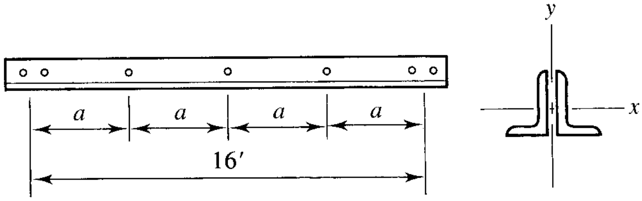
When the connectors are snug-tight bolts,

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_0^2 + \left(\frac{a}{r_i}\right)^2} \qquad \text{(AISC Equation E6-1)}$$

The column load tables for double-angle shapes show the number of intermediate connectors required for the given y-axis flexural-torsional buckling strength. The number of connectors needed for the x-axis flexural buckling strength must be determined from the requirement of Equation 4.14 that the slenderness of one angle between connectors must not exceed three-fourths of the overall slenderness of the double-angle shape.

Example 4.18 Compute the available strength of the compression member shown in Figure 4.22. Two angles, $5 \times 3 \times \frac{1}{2}$, are oriented with the long legs back-to-back ($2L5 \times 3 \times \frac{1}{2}$ LLBB) and separated by $\frac{3}{8}$ inch. The effective length *KL* is 16 feet, and there are three fully tightened intermediate connectors. A36 steel is used.

FIGURE 4.22



Solution Compute the flexural buckling strength for the *x*-axis:

$$\frac{K_x L}{r_x} = \frac{16(12)}{1.58} = 121.5$$

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(121.5)^2} = 19.39 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{36}} = 134$$

Since $\frac{KL}{r} < 4.71\sqrt{\frac{E}{F_y}}$, use AISC Equation E3-2.

$$F_{cr} = 0.658^{(F_y/F_e)}F_y = 0.658^{(36/19.39)}(36) = 16.55 \text{ ksi}$$

The nominal strength is

$$P_n = F_{cr}A_g = 16.55(2 \times 3.75) = 124.1 \text{ kips}$$

To determine the flexural-torsional buckling strength for the *y*-axis, use the modified slenderness ratio, which is based on the spacing of the connectors. The unmodified slenderness ratio is

$$\left(\frac{KL}{r}\right)_0 = \frac{KL}{r_y} = \frac{16(12)}{1.24} = 154.8$$

The spacing of the connectors is

$$a = \frac{16(12)}{4 \text{ spaces}} = 48 \text{ in.}$$

Then, from Equation 4.14,

$$\frac{Ka}{r_i} = \frac{Ka}{r_z} = \frac{48}{0.642} = 74.77 < 0.75(154.8) = 116.1 \quad \text{(OK)}$$

$$r_{ib} = r_y = 0.824 \text{ in.}$$

$$h = 2(0.746) + \frac{3}{8} = 1.867 \text{ in.}$$

$$\alpha = \frac{h}{2r_{ib}} = \frac{1.867}{2 \times 0.824} = 1.133$$

From AISC Equation E6-2, the modified slenderness ratio is

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_o^2 + 0.82\frac{\alpha^2}{(1+\alpha^2)}\left(\frac{a}{r_{ib}}\right)^2}$$

$$= \sqrt{(154.8)^2 + 0.82\frac{(1.133)^2}{\left[1+(1.133)^2\right]}\left(\frac{48}{0.824}\right)^2} = 159.8$$

This value should be used in place of KL/r_y for the computation of F_{cry}:

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29{,}000)}{(159.8)^2} = 11.21 \text{ ksi}$$

Since $\frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}} = 134$,

$$F_{cry} = 0.877F_e = 0.877(11.21) = 9.831 \text{ ksi}$$

From AISC Equation E4-3,

$$F_{crz} = \frac{GJ}{A_g\bar{r}_o^2} = \frac{11{,}200(2 \times 0.322)}{7.50(2.51)^2} = 152.6 \text{ ksi}$$

$$F_{cry} + F_{crz} = 9.831 + 152.6 = 162.4 \text{ ksi}$$

$$F_{cr} = \left(\frac{F_{cry}+F_{crz}}{2H}\right)\left[1 - \sqrt{1 - \frac{4F_{cry}F_{crz}H}{(F_{cry}+F_{crz})^2}}\right]$$

$$= \frac{162.4}{2(0.646)}\left[1 - \sqrt{1 - \frac{4(9.832)(152.6)(0.646)}{(162.4)^2}}\right] = 9.606 \text{ ksi}$$

The nominal strength is

$$P_n = F_{cr}A_g = 9.606(7.50) = 72.05 \text{ kips}$$

Therefore the flexural-torsional buckling strength controls.

LRFD Solution The design strength is

$$\phi_c P_n = 0.90(72.05) = 64.9 \text{ kips}$$

ASD Solution From Equation 4.7, the allowable stress is

$$F_a = 0.6F_{cr} = 0.6(9.606) = 5.764 \text{ ksi}$$

The allowable strength is

$$F_a A_g = 5.764(7.50) = 43.2 \text{ kips}$$

Answer Design compressive strength = 64.9 kips. Allowable compressive strength = 43.2 kips.

Example 4.19

Design a 14-foot-long compression member to resist a service dead load of 12 kips and a service live load of 23 kips. Use a double-angle shape with the short legs back-to-back, separated by ⅜-inch. The member will be braced at midlength against buckling about the x-axis (the axis parallel to the long legs). Specify the number of intermediate connectors needed (the midlength brace will provide one such connector). Use A36 steel.

LRFD Solution The factored load is

$$P_u = 1.2D + 1.6L = 1.2(12) + 1.6(23) = 51.2 \text{ kips}$$

From the column load tables, select 2L 3½ × 3 × ¼ SLBB, weighing 10.8 lb/ft. The capacity of this shape is 53.2 kips, based on buckling about the y-axis with an effective length of 14 feet. (The strength corresponding to flexural buckling about the x-axis is 63.1 kips, based on an effective length of 14/2 = 7 feet.) Note that this shape is a slender-element cross section, but this is taken into account in the tabular values.

Bending about the y-axis subjects the fasteners to shear, so a sufficient number of fasteners must be provided to account for this action. The table reveals that three intermediate connectors are required. (This number also satisfies Equation 4.14.)

Answer Use 2L 3½ × 3 × ¼ SLBB with three intermediate connectors within the 14-foot length.

ASD Solution The total load is

$$P_a = D + L = 12 + 23 = 35 \text{ kips}$$

From the column load tables, select 2L 3¼ × 3 × ¼ SLBB, weighing 10.8 lb/ft. The capacity is 35.4 kips, based on buckling about the y axis, with an effective length of

14 feet. (The strength corresponding to flexural buckling about the x axis is 42.0 kips, based on an effective length of $^{14}/_{2} = 7$ feet.) Note that this shape is a slender-element section, but this is taken into account in the tabular values.

Bending about the y axis subjects the fasteners to shear, so a sufficient number of fasteners must be provided to account for this action. The table reveals that three intermediate connectors are required. (This number also satisfies Equation 4.14.)

Answer Use 2L $3\frac{1}{2} \times 3 \times \frac{1}{4}$ SLBB with three intermediate connectors within the 14-foot length. ■

Connection Requirements for Built-Up Members Composed of Plates or Both Plates and Shapes

When a built-up member consists of two or more rolled shapes separated by a substantial distance, plates must be used to connect the shapes. AISC E6 contains many details regarding the connection requirements and the proportioning of the plates. Additional connection requirements are given for other built-up compression members composed of plates or plates and shapes.

Problems

AISC Requirements

4.3-1 Use AISC Equation E3-2 or E3-3 and determine the nominal axial compressive strength for the following cases:

a. $L = 10$ ft
b. $L = 30$ ft

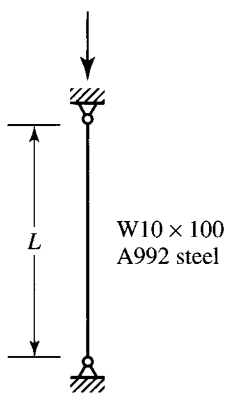


FIGURE P4.3-1

4.3-2 Compute the nominal axial compressive strength of the member shown in Figure P4.3-2. Use AISC Equation E3-2 or E3-3.

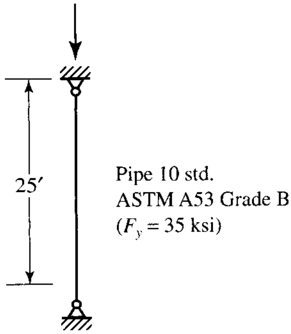


FIGURE P4.3-2

4.3-3 Compute the nominal compressive strength of the member shown in Figure P4.3-3. Use AISC Equation E3-2 or E3-3.

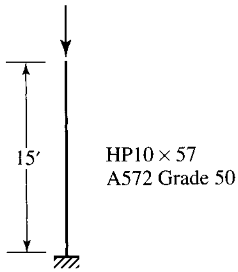


FIGURE P4.3-3

4.3-4 Determine the available strength of the compression member shown in Figure P4.3-4, in each of the following ways:

a. Use AISC Equation E3-2 or E3-3. Compute both the design strength for LRFD and the allowable strength for ASD.
b. Use Table 4-22 from Part 4 of the *Manual.* Compute both the design strength for LRFD and the allowable strength for ASD.

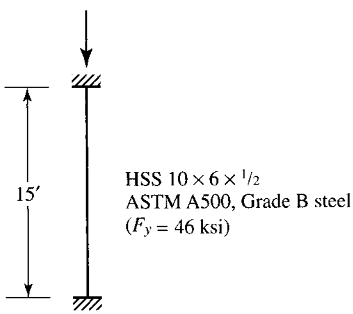


FIGURE P4.3-4

4.3-5 Determine the available axial compressive strength by each of the following methods:

a. Use AISC Equation E3-2 or E3-3. Compute both the design strength for LRFD and the allowable strength for ASD.
b. Use Table 4-22 from Part 4 of the *Manual.* Compute both the design strength for LRFD and the allowable strength for ASD.

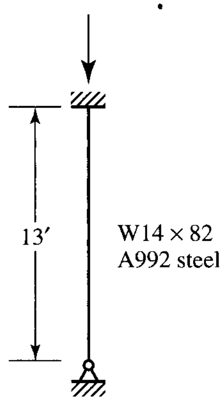


FIGURE P4.3-5

4.3-6 A W18 × 119 is used as a compression member with one end fixed and the other end pinned. The length is 12 feet. What is the available compressive strength if A992 steel is used?

a. Use AISC Equation E3-2 or E3-3. Compute both the design strength for LRFD and the allowable strength for ASD.
b. Use Table 4-22 from Part 4 of the *Manual.* Compute both the design strength for LRFD and the allowable strength for ASD.

4.3-7 An HSS 10 × 8 × 3⁄16 is used as a compression member with one end pinned and the other end fixed against rotation but free to translate. The length is 12 feet. Compute the nominal compressive strength for A500 Grade B steel ($F_y = 46$ ksi). *Note that this is a slender-element compression member, and the equations of AISC Section E7 must be used.*

4.3-8 A W21 × 101 is used as compression member with one end fixed and the other end free. The length is 10 feet. What is the nominal compressive strength if $F_y = 50$ ksi? *Note that this is a slender-element compression member, and the equations of AISC Section E7 must be used.*

4.3-9 Determine the maximum axial compressive service load that can be supported if the live load is twice as large as the dead load. Use AISC Equation E3-2 or E3-3.

a. Use LRFD.
a. Use ASD.

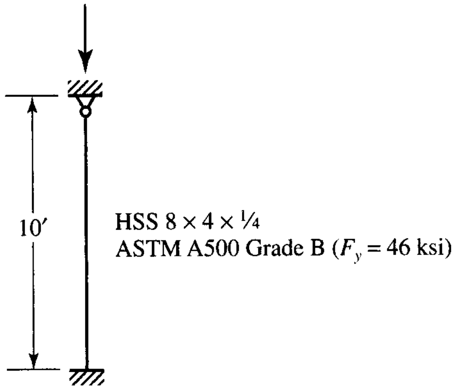


FIGURE P4.3-9

4.3-10 Determine whether the compression member shown in Figure P4.3-10 is adequate to support the given service loads.

a. Use LRFD.
b. Use ASD.

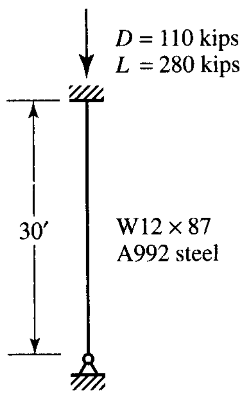


FIGURE P4.3-10

Design

4.6-1 a. Select a W14 of A992 steel. Use the column load tables.

1. Use LRFD.
2. Use ASD.

b. Select a W16 of A992 steel. Use the trial-and-error approach covered in Section 4.6.

1. Use LRFD.
2. Use ASD.

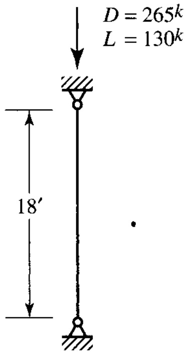


FIGURE P4.6-1

4.6-2 A 20-foot long column is pinned at the bottom and fixed against rotation but free to translate at the top. It must support a service dead load of 110 kips and a service live load of 110 kips.

a. Select a W12 of A992 steel. Use the column load tables.
   1. Use LRFD.
   2. Use ASD.
b. Select a W18 of A992 steel. Use the trial-and-error approach covered in Section 4.6.
   1. Use LRFD.
   2. Use ASD.

4.6-3 Select a rectangular (not square) HSS ($F_y = 46$ ksi).

a. Use LRFD.
b. Use ASD.

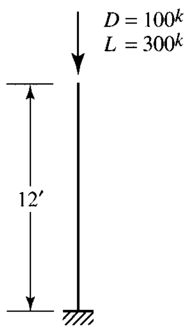


FIGURE P4.6-3

4.6-4 Select a steel pipe of A53 Grade B steel ($F_y = 35$ ksi). Specify whether your selection is Standard, Extra-Strong, or Double-Extra Strong.

a. Use LRFD.
b. Use ASD.

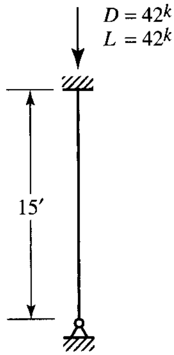


FIGURE P4.6-4

4.6-5 Select an HP-shape for the conditions of Problem 4.6-3. Use $F_y = 50$ ksi.

a. Use LRFD.
b. Use ASD.

4.6-6 Select a rectangular (not square) HSS for the conditions of Problem 4.6-4.

a. Use LRFD.
b. Use ASD.

4.6-7 For the conditions shown in Figure P4.6-7, use LRFD and

a. select a W12 of A992 steel.
b. select a steel pipe.
c. select a square HSS.
d. select a rectangular HSS.

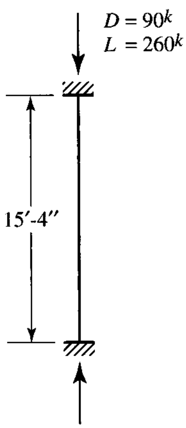


FIGURE P4.6-7

4.6-8 Same as Problem 4.6-7, but use ASD.

4.6-9 For the conditions shown in Figure P4.6-7, select an A992 W-shape with a nominal depth of 21 inches.

a. Use LRFD.
b. Use ASD.

Effective Length

4.7-1 A W16 × 100 with $F_y = 60$ ksi is used as a compression member. The length is 13 feet. Compute the nominal strength for $K_x = 2.1$ and $K_y = 1.0$.

4.7-2 An HSS 10 × 6 × 5/16 with $F_y = 46$ ksi is used as a column. The length is 15 feet. Both ends are pinned, and there is support against weak axis buckling at a point 6 feet from the top. Determine

a. the design strength for LRFD.
b. the allowable *stress* for ASD.

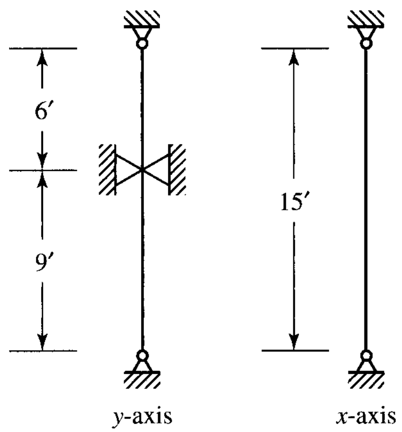


FIGURE P4.7-2

4.7-3 A W12 × 79 of A572 Grade 60 steel is used as a compression member. It is 28 feet long, pinned at each end, and has additional support in the weak direction at a point 12 feet from the top. Can this member resist a service dead load of 180 kips and a service live load of 320 kips?

a. Use LRFD.
b. Use ASD.

4.7-4 Use A992 steel and select a W14 shape for an axially loaded column to meet the following specifications: The length is 22 feet, both ends are pinned, and there is bracing in the weak direction at a point 10 feet from the top. The service dead load is 142 kips, and the service live load is 356 kips.

a. Use LRFD.
b. Use ASD.

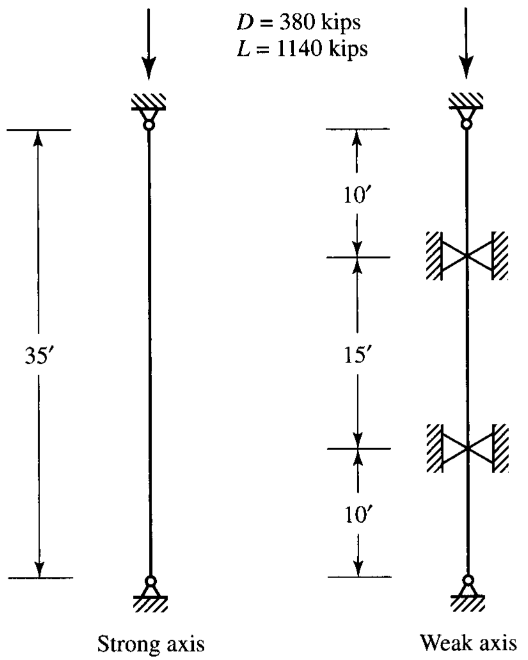


FIGURE P4.7-5

4.7-5 Use A992 steel and select a W shape.

a. Use LRFD.
b. Use ASD.

4.7-6 Select a rectangular (not square) HSS for use as a 15-foot-long compression member that must resist a service dead load of 35 kips and a service live load of 80 kips. The member will be pinned at each end, with additional support in the weak direction at midheight. Use A500 Grade B steel ($F_y = 46$ ksi).

a. Use LRFD.
b. Use ASD.

4.7-7 Select the best rectangular (not square) HSS for a column to support a service dead load of 33 kips and a service live load of 82 kips. The member is 27 feet long and is pinned at the ends. It is supported in the weak direction at a point 12 feet from the top. Use $F_y = 46$ ksi.

a. Use LRFD.
b. Use ASD.

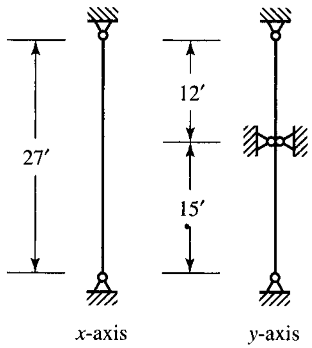


FIGURE P4.7-7

4.7-8 The frame shown in Figure P4.7-8 is unbraced, and bending is about the x-axis of the members. All beams are W18 × 35, and all columns are W10 × 54.

a. Determine the effective length factor K_x for column AB. Do not consider the stiffness reduction factor.
b. Determine the effective length factor K_x for column BC. Do not consider the stiffness reduction factor.
c. If $F_y = 50$ ksi, is the stiffness reduction factor applicable to these columns?

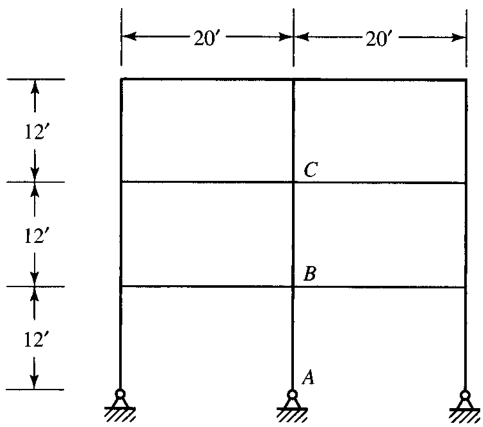


FIGURE P4.7-8

4.7-9 The given frame is unbraced, and bending is about the x axis of each member. The axial dead load supported by column AB is 204 kips, and the axial live load is 408 kips. $F_y = 50$ ksi. Determine K_x for member AB. Use the stiffness reduction factor if possible.

a. Use LRFD.
b. Use ASD.

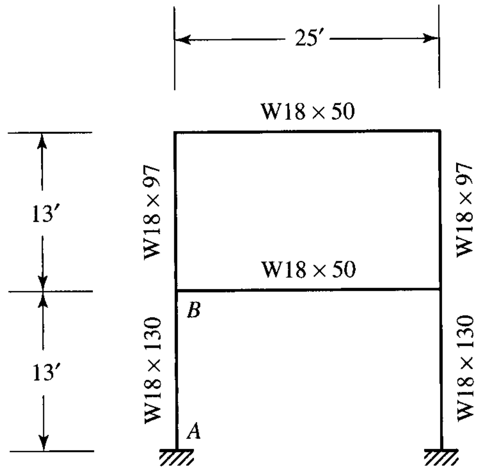


FIGURE P4.7-9

4.7-10 The rigid frame shown in Figure P4.7-10 is unbraced. The members are oriented so that bending is about the strong axis. Support conditions in the direction perpendicular to the plane of the frame are such that $K_y = 1.0$. The beams are W18 × 50, and the columns are W12 × 72. A992 steel is used. The axial compressive dead load is 50 kips, and the axial compressive live load is 150 kips.

a. Determine the axial compressive design strength of column *AB*. Use the stiffness reduction factor if applicable.
b. Determine the allowable axial compressive strength of column *AB*. Use the stiffness reduction factor if applicable.

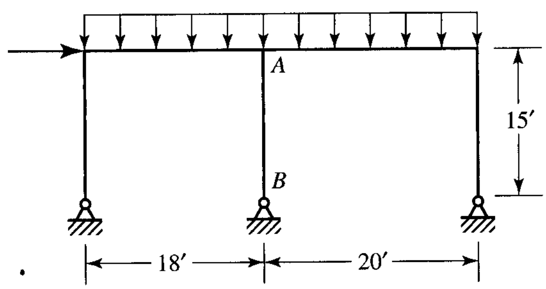


FIGURE P4.7-10

4.7.11 The frame shown in Figure P4.7-11 is unbraced against sidesway. Relative moments of inertia of the members have been assumed for preliminary design purposes. Use the alignment chart and determine K_x for members *AB, BC, DE,* and *EF*.

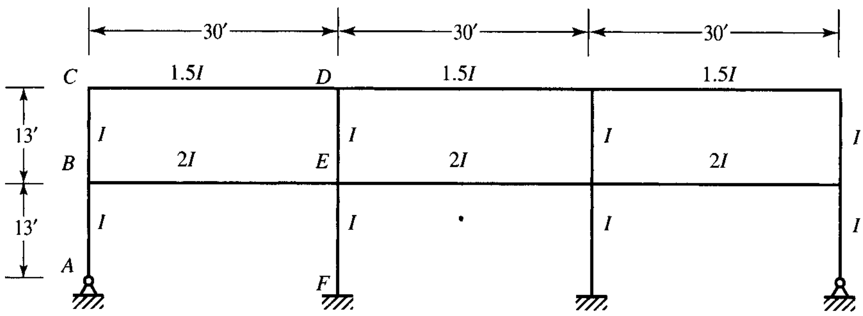


FIGURE P4.7-11

4.7-12 The frame shown in Figure P4.7-12 is unbraced against sidesway. Assume that all columns are W14 × 61 and that all girders are W18 × 76. ASTM A992 steel is used for all members. The members are oriented so that bending is about the x-axis. Assume that $K_y = 1.0$

a. Use the alignment chart to determine K_x for member *GF*. Use the stiffness reduction factor if applicable. For member *GF*, the service dead load is 80 kips and the service live load is 159 kips.
b. Compute the nominal compressive strength of member GF.
c. Estimate K_x from Table C-C2.2 in the Commentary and compare your estimate with the results of part (a).

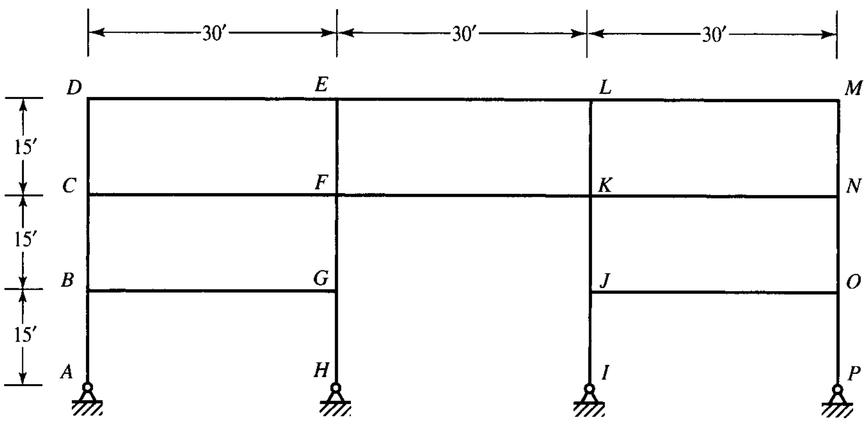


FIGURE P4.7-12

4.7-13 The frame shown in Figure P4.7-13 is unbraced against sidesway. The columns are HSS 6 × 6 × ⅝, and the beams are W12 × 22. ASTM A500 grade B steel ($F_y = 46$ ksi)

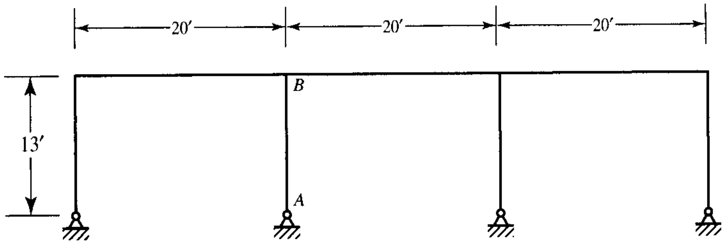


FIGURE P4.7-13

is used for the columns, and $F_y = 50$ ksi for the beams. The beams are oriented so that bending is about the x-axis. Assume that $K_y = 1.0$.

a. Use the alignment chart to determine K_x for column AB. Use the stiffness reduction factor if applicable. For column AB, the service dead load is 17 kips and the service live load is 50 kips.
b. Compute the nominal compressive strength of column AB.

4.7-14 The rigid frame shown in Figure P4.7-14 is unbraced in the plane of the frame. In the direction perpendicular to the frame, the frame is braced at the joints. The connections at these points of bracing are simple (moment-free) connections. Roof girders are W14 × 30, and floor girders are W16 × 36. Member BC is a W10 × 45. Use A992 steel and select a W-shape for AB. Assume that the controlling load combination causes no moment in AB. The service dead load is 25 kips and the service live load is 75 kips.

a. Use LRFD.
b. Use ASD.

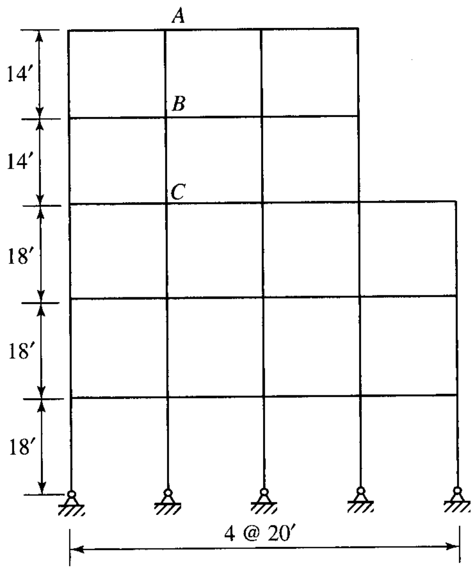


FIGURE P4.7-14

Torsional and Flexural-Torsional Buckling

4.8-1 Compute the nominal compressive strength for a WT10.5 × 91 with an effective length of 18 feet with respect to each axis. Use A992 steel and the procedure of AISC Section E4 (not the column load tables).

4.8-2 Use A572 Grade 50 steel and compute the nominal strength of the column shown in Figure P4.8-2. The member ends are fixed in all directions (x, y, and z).

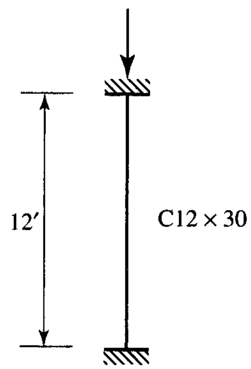


FIGURE P4.8-2

4.8-3 Select a WT section for the compression member shown in Figure P4.8-3. The load is the total service load, with a live-to-dead load ratio of 2.5:1. Use $F_y = 50$ ksi.

a. Use LRFD.
b. Use ASD.

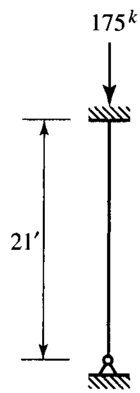


FIGURE P4.8-3

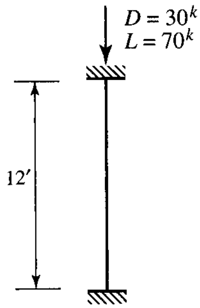


FIGURE P4.8-4

4.8-4 Select an American Standard Channel for the compression member shown in Figure P4.8-4. Use A36 steel. The member ends are fixed in all directions (x, y, and z).

a. Use LRFD.
b. Use ASD.

Built-Up Members

4.9-1 Verify the value of r_y given in Part 1 of the *Manual* for the double-angle shape 2L5 × 3½ × ½ LLBB. The angles will be connected to a ⅜-inch-thick gusset plate.

4.9-2 Verify the values of y_2, r_x, and r_y given in Part 1 of the *Manual* for the combination shape consisting of a W12 × 26 with a C10 × 15.3 cap channel.

4.9-3 A column is built up from four 6 × 6 × ⅝ angle shapes as shown in Figure P4.9-3. The plates are not continuous but are spaced at intervals along the column length and function to maintain the separation of the angles. They do not contribute to the cross-sectional properties. Compute r_x and r_y.

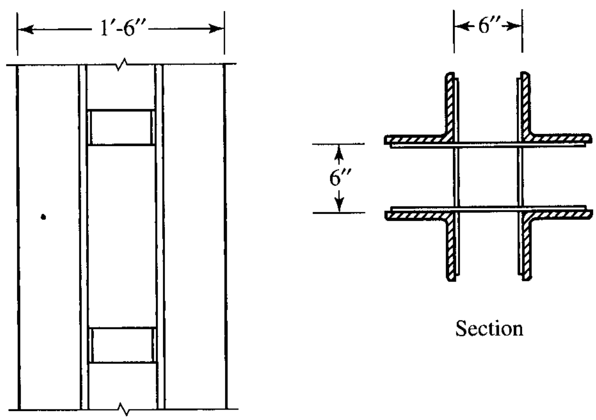


FIGURE P4.9-3

4.9-4 An unsymmetrical compression member consists of ½ × 12 top flange, a ½ × 7 bottom flange, and a ⅜ × 16 web (the shape is symmetrical about a vertical centroidal axis). Compute the radius of gyration about each of the principal axes.

4.9-5 A column for a multistory building is fabricated from ASTM A588 plates as shown in Figure P4.9-5. Compute the nominal axial compressive strength based on flexural buckling (do not consider torsional buckling). Assume that the components of the cross section are connected in such a way that the section is fully effective.

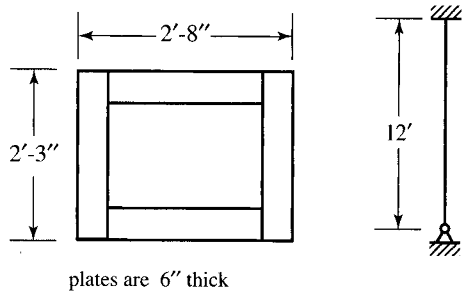


FIGURE P4.9-5

4.9-6 Compute the nominal compressive strength based on flexural buckling for the built-up shape shown in Figure P4.9-6 (do not consider torsional buckling). Assume that the components of the cross section are connected in such a way that the section is fully effective. ASTM A242 steel is used.

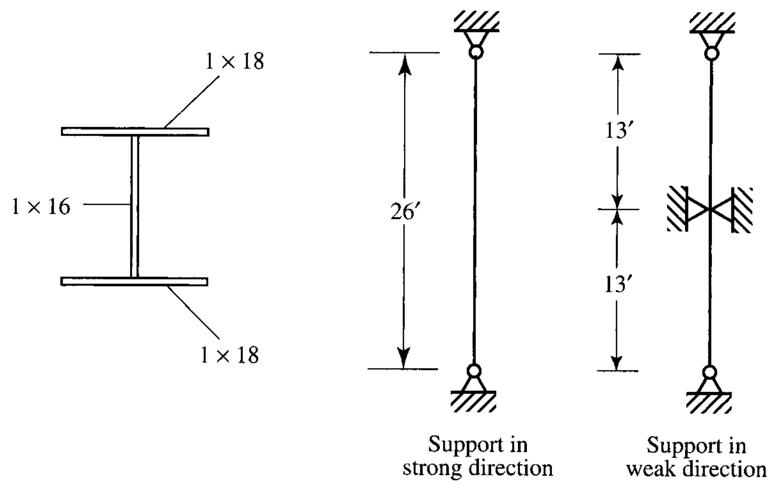


FIGURE P4.9-6

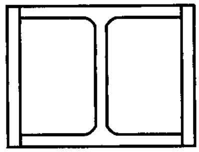


FIGURE P4.9-7

4.9-7 Two plates $\frac{9}{16} \times 10$ are welded to a W10 × 49 to form a built-up shape, as shown in Figure P4.9-7. Assume that the components are connected so that the cross section is fully effective. $F_y = 50$ ksi, and $K_xL = K_yL = 25$ ft.

a. Compute the nominal axial compressive strength based on flexural buckling (do not consider torsional bucking).
b. What is the percentage increase in strength from the unreinforced W10 × 49?

4.9-8 A structural tee shape is fabricated by splitting an HP14 × 117, as shown in Figure P4.9-8. Compute the nominal axial compressive strength based on flexural buckling (do not consider flexural-torsional buckling). Account for the area of the fillets at the web-to-flange junction. A572 Grade so steel is used. The effective length is 10 feet with respect to both axes.

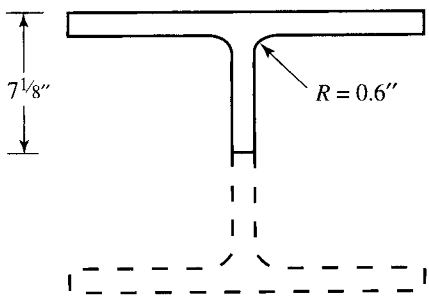


FIGURE P4.9-8

4.9-9 A column cross section is built up from four L5 × 5 × $\frac{3}{4}$, as shown in Figure P4.9-9. The angles are held in position by lacing bars, whose primary function is to hold the angles in position. The lacing is not considered to contribute to the cross-sectional area, which is why it is shown by dashed lines. AISC Section E6 covers the design of lacing. The effective length is 30 feet with respect to both axes, and A572 Grade 50 steel is used. Investigate flexural buckling only (no torsional buckling) and compute

a. the design strength for LRFD.
b. the allowable strength for ASD.

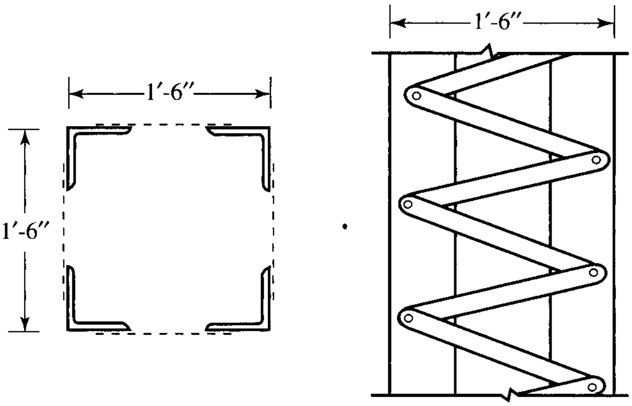


FIGURE P4.9-9

4.9-10 Compute the available strengths (for both LRFD and ASD) for the following double-angle shape: 2L6 × 4 × ⅝, long legs ⅜-inch back-to-back, $F_y = 50$ ksi. The effective length KL is 18 feet for all axes, and there are two intermediate fully tightened bolts. Use the procedure of AISC E4 (not the column load tables). Compare the flexural and the flexural-torsional buckling strengths.

4.9-11 For the conditions shown in Figure P4.9-11, select a double-angle section (⅜-inch gusset plate connection). Use A36 steel. Specify the number of intermediate connectors.

a. Use LRFD.
b. Use ASD.

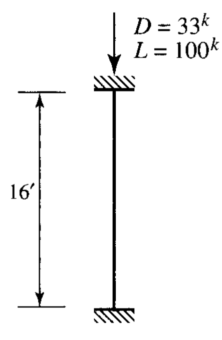


FIGURE P4.9-11

4.9-12 Use LRFD and select a double-angle shape for the top chord of the truss of Problem 3.8-2. Use $K_x = K_y = 1.0$. Assume ⅜-inch gusset plates, and use A36 steel.